On the Recovery of Iron from Overstrain

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PHILOSOPHICAL TRANSACTIONS.

I. *On the Recovery of Iron from Overstrain.*

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IT has long been known that iron which has been overstrained in tension—that is to say strained beyond the yield-point so that it suffers a permanent stretch—possesses very different elastic properties from the same iron in its primitive condition. The material is said to be "hardened" by stretching,* since the ultimate effect of such treatment is to raise the elastic limit and reduce the ductility of the material.

More recently, attention has been called to the fact that, primarily, the result of tensile overstrain is to make iron assume a semi-plastic state, so that the elastic limit, instead of being raised by stretching, is first of all lowered, it may be to zero.† This plasticity may be shown by applying a comparatively small load to a bar of iron or steel which has just been overstrained by the application and removal of a large stretching load. When the small load is put on, the bar will be found to elongate further than it would had the material been in its primitive state; and a slight continued elongation—a "creeping"—may occur after the small load has been applied. If this load be withdrawn, a quite appreciable permanent, or semi-permanent, set will be found to have been produced; a set which diminishes slightly,

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* EWING, "On Certain Effects of Stress," 'Proc. Roy. Soc.,' No. 205, 1880. The raising of the elastic limit due to stretching seems to have been first noted in 1865 by THALÉN. See a translation of his paper in the 'Phil. Mag.' for September of that year.

† BAUSCHINGER, "Ueber die Veränderung der Elasticitätsgrenze," 'Civilingenieur,' 1881, or "Mittheilungen aus dem mechanisch-technischen Laboratorium der K. Polytechnischen Schule in München." An account of BAUSCHINGER'S work is given in UNWIN'S book on "Testing of Materials of Construction."

EWING, "On Measurements of Small Strains in the Testing of Materials and Structures," 'Proc. Roy. Soc.,' vol. 58, April, 1895.

and, if small, may vanish, provided time be allowed for backward creeping to take effect. It may also be shown that, if the re-applied load be increased, the elongation produced will increase in a greater proportion. Thus, if a stress-strain curve be obtained from a recently overstrained bar of iron or steel, it will show, even for small loads, a marked falling away from the straight line which would indicate obedience to HOOKE's law.

It is the recovery from this semi-plastic state induced by overstrain to a condition of perfect or nearly perfect elasticity with raised elastic limit, that is referred to in the title of this paper. Such recovery is known to be effected by mere lapse of time,* and the object of the experiments about to be described is to show the effect of moderate temperature, of mechanical vibration, and of magnetic agitation, on this slow return to the elastic state, and further to illustrate this recovery by means of compression tests. One section of the paper deals with the phenomenon of hysteresis in the relation of extension to stress, which is exhibited in a marked degree by iron in the overstrained state. Incidentally, attention will be called to subsidiary points of interest.

The experiments were carried out in the Engineering Laboratory of Cambridge University, and were the outcome of suggestions by Professor EWING. It was on his suggestion that the effect of moderate temperature on recovery from overstrain was tried, and the result of that trial led to much of the work incorporated in this paper.

Before going into details of the experiments it may be of interest to give, drawn to a small scale, an ordinary complete stress-strain diagram, such as is obtained in the testing of iron or steel. The period in the history of iron subjected to tensile stress which is about to be investigated, may thus be more clearly indicated. The curve given in Diagram No. I. was sketched by hand, roughly, from data obtained from the experiments which will be described later. It applies to steel not previously submitted to overstrain.

For the portion *ab* of this curve HOOKE's law is obeyed. At *b* the yield-point occurs, and as soon as this point is passed the material becomes overstrained. During the large yielding which takes place at the yield-point the load may be reduced without causing the extension to stop. After stretching by a large amount as compared with elastic extension, the material will be found to have hardened; so that to produce further yielding the load must be increased. The stress-strain diagram may now be represented by some such curve as *cd*. If at *d* the load be removed, and at once gradually replaced, then the stress-strain curve may follow a path such as *de*, *ef*. These curves *de* and *ef*, when obtained in such a manner that the exten-

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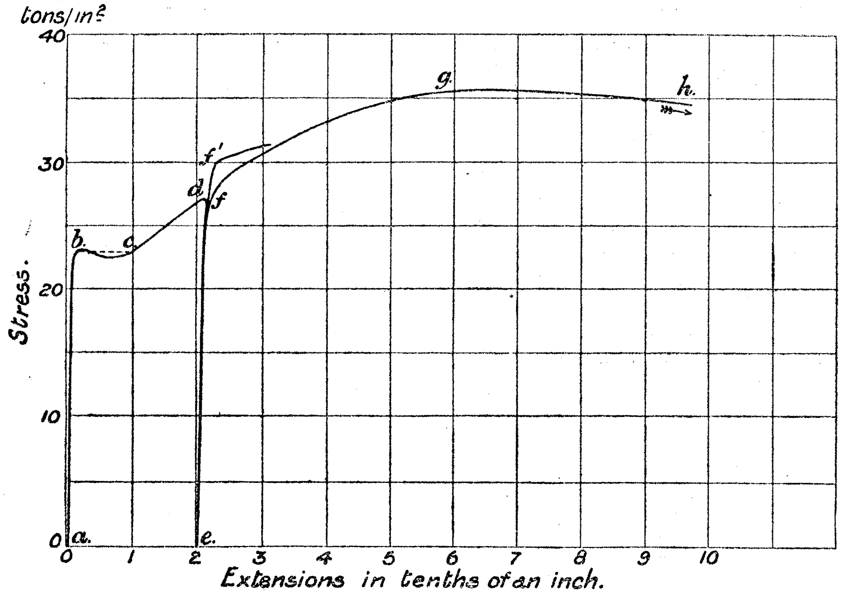
* BAUSCHINGER, 'DINGLER's Journal,' vol. 224, p. 5, and papers already cited.

EWING, both papers cited above.

Reference might also be made to Lord KELVIN's discovery of the effect of a Sunday's rest on wires which had been subjected to torsional vibrations throughout the preceding week.—See article, "Elasticity," 'Encycl. Brit.'

sions can be plotted to a much larger scale, show the imperfect elasticity of recently overstrained iron which has been referred to above; that is, they show that the material is semi-plastic. If time be allowed to elapse between unloading and reloading, the recovery from the effect of overstrain may be shown in a diagram like the present, by some such curve as *ef′*. When no interval of time is allowed to elapse between the removal and the replacement of the load, then the stress-strain curve is continued in the manner shown by *fg*, until a point *g* is reached, at which local extension sets in. When this happens the stress may be diminished, and fracture may take place at a load lower than that at which local extension occurred. The stress per square inch of fractured area is, however, found to be much greater than the stress per square inch of the actual area when local elongation began.

Diagram No. I.



The Apparatus and the Material.

The straining and testing in the following experiments was done by means of the 50-ton Wicksteed single-lever hydraulic testing machine of the Cambridge Engineering Laboratory. With this machine the magnitude of the load applied could be read in tons to a second decimal place by means of a vernier, and to a third decimal place roughly by estimation. Thus a load could be applied accurately if necessary to, it may be said, $\frac{1}{500}$th of a ton.

The small strains of extension were measured by Professor Ewing's extensometer.*

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* For a full description of this instrument see the paper already cited "On the Measurement of Small Strains, &c.," 'Proc. Roy. Soc.,' vol. 58, April, 1895.

This instrument gave the extension occurring in the middle 8 inches of the length of the specimen under test. It enabled elongations to be measured to $\frac{1}{50000}$th of an inch, and to be measured to that degree of precision with ease and confidence. The instrument was found especially convenient on account of the facility with which it could (by the aid of a distance-piece) be immediately re-applied to a specimen which had just been strained beyond its yield-point; and also on account of the readiness with which the correct adjustment of the instrument itself could be tested.

The specimens employed were, with one or two exceptions, 18 to 20-inch lengths of steel-rod, 1 inch in diameter, of a quality which may be described as semi-mild. The details of the particular rods employed in the various experiments will be given when these come to be described. Here, as illustrating the general character of the material, the chemical analyses and elastic characteristics of two of the bars made use of will be given. The first is that from which diagram No. IX. has been obtained. A specimen from it showed a well-defined yield-point at a stress of 23 tons to the square inch, and gave an ultimate strength of 36½ tons per square inch of original area, with an elongation of 22½ per cent. on an 8-inch length. The second bar is that from which diagrams Nos. IV. and VII. have been obtained; it was characterised by a small flaw running up the centre through the whole length of the bar. A well-defined yield-point was not obtained with specimens from this bar; there was a distinct departure from obedience to HOOKE's law, at a stress of about 22 tons per square inch, but the yield-point should, perhaps, be placed as much as 6 or 7 tons higher than this stress, at which elastic behaviour broke down.* The ultimate strength of the material as obtained from a short specimen was 39 tons per square inch, the elongation being only about 20½ per cent. on a 3-inch length. The chemical analyses of these two bars were kindly supplied by Messrs. EDGAR ALLEN and Co., Sheffield, from whom the material was obtained; they are as follows:—

	Bar of diagram No. IX.	Bar of diagrams Nos. IV. and VII.
Carbon	0·430	0·450
Silicon	0·112	0·093
Sulphur	0·010	0·012
Phosphorus	0·016	0·021
Manganese	0·450	0·410
Iron (by difference)	98·982	99·014
	100·000	100·000

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[*] See the first column of the table on p. 15.

The Method of Experimenting.

The general procedure adopted in experimenting will now be described. First, the diameter of the specimen was determined from the mean of ten micrometer readings, taken at five equidistant places along the 8-inch length to which the extensometer was to be applied. For example, the following readings were obtained for a certain unturned specimen :—

$$\text{Diameter} = \left\{\begin{array}{rrrrr} 1''\cdot0069 & 71 & 65 & 64 & 61 \\ 70 & 71 & 71 & 59 & 54 \end{array}\right\} = 1''\cdot0066.$$

Not only was this done for virgin specimens, but whenever a yield-point had been passed, the diameter was re-determined by means of fresh readings. For example, the specimen already instanced was subjected to a pull gradually increasing to 35 tons to the square inch of original section, the yield-point occurring at 23 tons per square inch. After the removal of this large stretching load the new diameter was determined from the following readings :—

$$\text{Diameter} = \left\{\begin{array}{rrrrr} 0''\cdot9918 & 21 & 18 & 20 & 25 \\ 14 & 22 & 24 & 15 & 24 \end{array}\right\} = 0\cdot9920 \text{ of an inch.}$$

After the determination of the diameter of a specimen at each stage of an experiment, a table was formulated, from which the total load applied could be translated into tons per square inch of section, the stress being in every case measured with reference to the section at the beginning of each separate test.

These preliminaries having been completed, the specimen was put into the testing machine, the extensometer was attached, and the load was gradually applied. Extensometer readings were taken sometimes only after the addition of every four tons of stress, sometimes after each ton, sometimes after each half or quarter ton, according to judgment.

The following two series of readings are given for a typical experiment; they will serve to explain the usual procedure. The first series shows the elastic properties of a certain virgin specimen; the second shows the plastic nature of the same specimen immediately after the overstrain produced by the primary loading.

Stress in tons per square inch.	Total load in tons.	Extensometer readings. Unit = $\frac{1}{50000}$ of an inch.	Differences.
0	0	0	
1	0·79	30	30
2	1·58	60	30
4	3·16	120	60
6	&c.	180	60
8	...	240	60
10	...	300	60
12	...	360	60
14	...	421	61
16	...	481	60
18	...	540	59
20	...	600	60
22	...	660	60
24	...	720	60
25	...	750	30
26	...	780	30
27	...	810	30
		still 810 after 4 minutes, but 830 ,, 6 ,, &c. &c. (see Curve No. 1, Diagram II.) 1595 after 20 minutes	
$27\frac{1}{2}$	...	1700 and then quickly out of range	
30	put on gradually and kept on for 3 minutes, the beam of the testing machine remaining steady		

These figures show that this specimen has accurately obeyed HOOKE'S law, until a stress of 27 tons per square inch was attained. At this load the yield-point was expected to occur, and although the extensometer reading obtained gave no evidence of the proximity of such a point, by simply allowing the load to remain on for a short time (4 minutes) the creeping recorded above set in, or perhaps spread from without to within the 8-inch length under test.* The yield-point, with this specimen, has, therefore, coincided with the elastic limit as accurately as the extensometer can measure. Usually, in testing, imperfection of elasticity is shown before the yield-point is reached, and if a load less than that at the yield-point be allowed to act for some time, then a slight creeping probably supervenes. When, however, a bar, like that referred to above, has shown very perfect elasticity up to the yield-point, it is probable that a load very little under that at which the yield-point occurs could be sustained for an indefinite time without creeping taking place. Even although slight imperfection of elasticity be shown before the yield-point, experiments showed that no creeping need necessarily occur for a pause in the loading of at least a night's duration.

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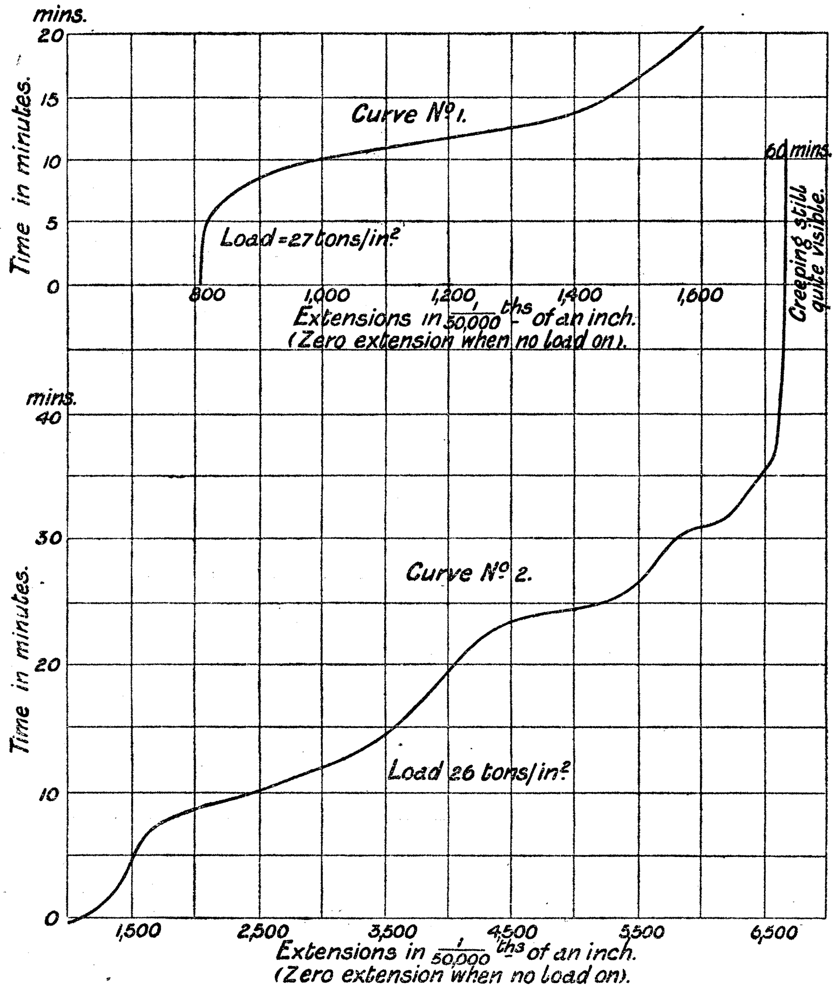
* The fact that yielding takes some time to start has already been recorded by Professor EWING in his paper cited above, "On Measurements of Small Strains, &c." (see pp. 135 and 136). He has noticed that yielding may begin in a part of the bar lying outside the 8-inch length to which the extensometer is applied, and may gradually spread along the bar.

The manner in which yielding under a constant load proceeds after the yield-point has just been passed is often very irregular. The curves given on Diagram II. illustrate this yielding with time for two entirely different specimens. The first shows the creeping referred to above as having started 4 minutes after the application of the load which was its cause. The second curve shows a larger yielding of much longer duration. It occurred under a load of 26 tons per square inch, but before this load was attained—when 25 and 25½ tons per square inch were acting—considerable creeping had already taken place. The readings from which these curves have been obtained were taken at intervals of one minute.

Diagram No. II.—(Manner in which yielding occurs at the yield-point.)



To return to the table of figures given above, the maximum load (of 30 tons to the square inch of original section) was found, after its removal, to have produced a permanent set of 0·22 of an inch on the 8-inch length; this corresponds to an extensometer reading of 11,000; such a reading is, of course, far beyond the range of the extensometer. Immediately after the maximum load had been removed, the diameter of the specimen was re-measured and the reduced section determined. The extensometer was then re-applied, the specimen was re-loaded, and the following readings observed :—

Tons per sq. in. (of reduced section).	Extensometer readings.	Differences.
0	0	
1 (= 0·77 ton of total load)	30	30
2	61	31
4	125	64
6	190	65
8	260	70
10	329	69
12	399	70
14	469	70
16	539	70
18	613	74
20	687	74
22	764	77
24	845	81
26	930	85
28	1028	98
30	1150	122

The load was now removed, and during its removal the following three readings were taken :

Tons/inch².	Extensometer.
20	830
10	477
0	60 but diminishing slightly with lapse of time

The series of increasing differences shown in this second table plainly indicates a change in the elastic state of the material. HOOKE's law is no longer obeyed.

This augmentation of the differences is, to some extent, associated with creeping, and to a greater extent, the higher is the applied load. Thus it is essential, if consistent results are to be obtained, that the interval of time which elapses between successive readings should always be kept the same. If a pause had been allowed to occur after the addition of any of the higher loads in this second table, then, owing to prolonged creeping, a larger difference would have been obtained than is recorded above. On proceeding with the loading, however, the immediately succeeding difference, or differences, would have been smaller than according to the table. For had there been no interruption, part of the creeping which occurred during the pause would have been recorded on the addition of the subsequent loads.

In experiments on a virgin piece of which the first table given above is typical, the time element does not enter, for there is no perceptible creeping until the yield-point is all but reached.

Slow Recovery of Elasticity with lapse of Time.

Before proceeding to describe the effect of special treatments on recovery from tensile overstrain, I give two instances of the slow recovery from overstrain with lapse of time, similar to the examples already given by Professor EWING.*

The curves in Diagram No. IIIA. illustrate this slow recovery for a specimen of 1 inch round steel rod, which has been strained to or very little beyond its yield-point. The material had an ultimate strength of 37 tons per square inch of original area, the total elongation being almost 23 per cent. on an 8-inch length. The yield-point was well defined and occurred at a stress slightly under 27 tons to the square inch. The readings from which the various curves have been plotted are given in the table on p. 11. The curves were obtained in the usual manner, the stresses being plotted as ordinates, and the corresponding extensions as abscissæ.

Curve No. 1 of Diagram IIIA. is a record of the primary test of the specimen. It shows that HOOKE's law has been obeyed up to a load of 26 tons to the square inch; and that before 27 tons there was a well-marked yield-point. This load of 27 tons per square inch was kept on for two minutes, by which time rapid stretching had ceased, as was shown by the beam of the testing machine remaining stationary. There was still a slow creeping, which probably would have continued for hours or days, becoming however slower and slower. Curve No. 2 represents a test performed as shortly after the removal of this load as the remeasurement of the diameter and the calculation of the reduced area would permit: it illustrates the semi-plastic condition of the material immediately after overstrain. In the plotting of this, and all subsequent curves in the diagram, it will be noticed that the origin for the measurement of extensions has been displaced; this was merely to keep the curves distinct and to facilitate comparison.

Curves Nos. 3, 4, 5, and 6, obtained at succeeding intervals, illustrate the gradual recovery of the elasticity lost by the overstrain. This recovery will be noticed to be quickest at first, and latterly to be very slow. In Curves 3, 4, and 5 the load was not allowed to exceed 27 tons per square inch. Curve No. 6 shows the recovery to have been nearly, though not quite, perfect after the material had been allowed to rest for 6 days 3 hours. In this test the load was gradually increased beyond the 27 tons, and a new yield-point was not obtained till rather over 30 tons to the square inch was reached.

Curve No. 7 shows the plastic nature of the material immediately after this second overstrain, and No. 8 the condition after 4 days' rest. Thus after 4 days the recovery

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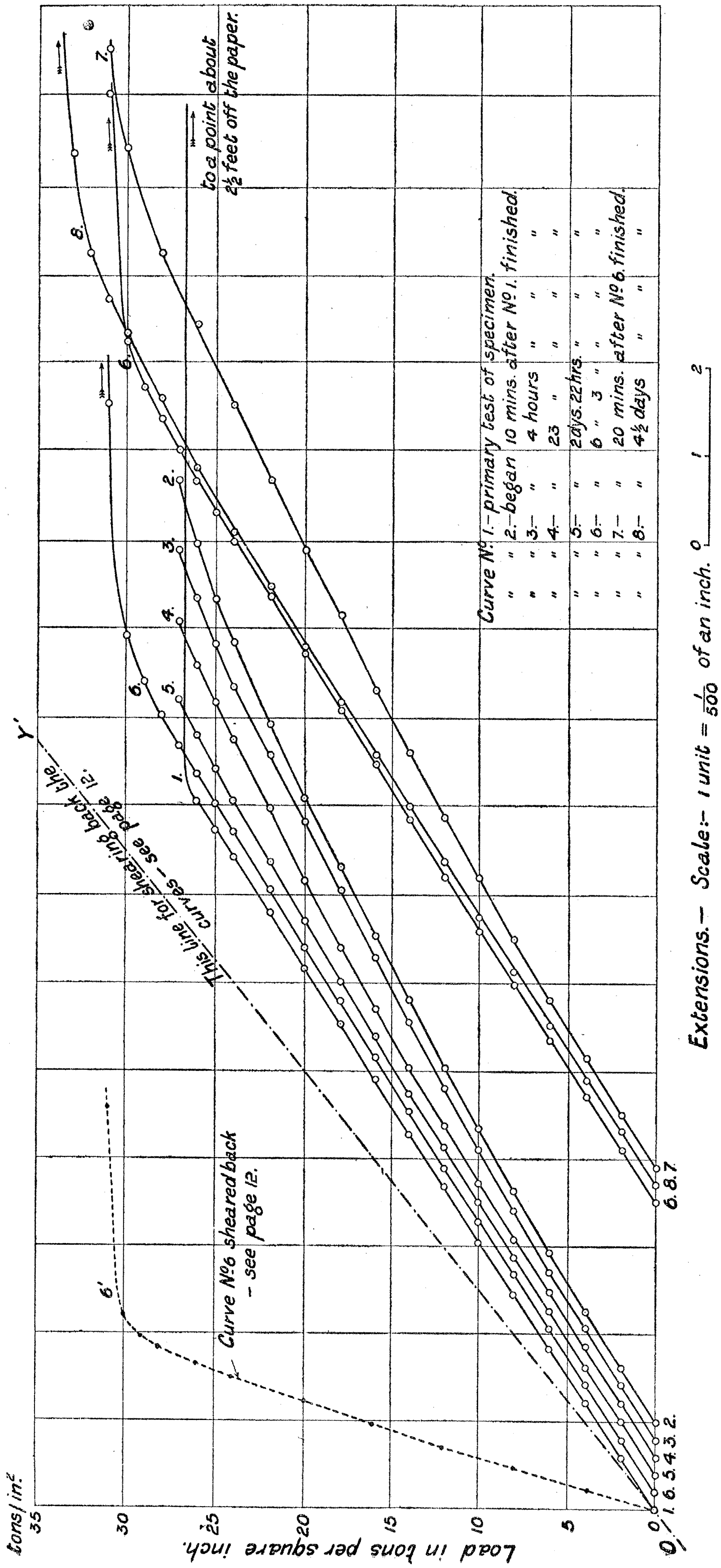
* See page 139, &c., of his paper "On Measurements of Small Strains, &c."

Diagram No. IIIA.—(Slow recovery with lapse of time.)



was by no means perfect, and on increasing the load beyond the 30 tons an almost immediate falling away was observed, and (as is shown in the curve) a third, rather indefinite, yield-point was indicated. Further yield-points might have been obtained had this treatment been proceeded with.

TABLE giving readings for Diagram No. III. (Slow recovery with time).

Load in tons/in².	Extensometer readings for various curves of Diagram III.							
	No. 1. Zero time.	No. 2. 10 mins.	No. 3. 4 hours.	No. 4. 23 hours.	No. 5. 2 days 22 hours.	No. 6. 6 days 3 hours.	No. 7. 20 mins. (No. 6, zero.)	No. 8. 4 days (No. 6, zero.)
0	0	0	0	0	0	0	0	0
2	60	62	62	61	61	60	61	60
4	121	125	128	125	122	122	127	120
6	182	192	191	188	186	186	191	181
8	243	262	260	249	248	249	260	243
10	304	333	330	311	310	309	330	305
12	368	405	401	378	372	370	397	368
14	428	481	477	444	434	433	471	429
16	491	553	550	511	500	498	542	489
18	553	631	625	581	562	560	626	549
20	618	709	701	658	630	621	700	610
22	680	793	779	738	698	687	778	678
24	742	883	855	817	765	750	860	740
25	772	933	902	859	801	781		
26	803	999	953	900	841	816	942	810
27	Out of range	1068 to 1117 in 3 mins.	1010 to 1035 in 3 mins.	949 to 978 in 3 mins.	881 to 897 in 3 mins.	850		
28	...	...	...	...	...	884	1035	889
29	...	...	...	...	...	921		
30	...	...	...	...	...	972	1152	964
31	...	...	...	...	...	1251 and then very large yielding	1262 to 1382 in 2 mins.	1004
32	...	...	...	...	...	...	...	1055
33	...	...	...	...	...	...	...	1166
34	...	...	...	...	...	...	...	Very large yielding
Load removed. 0	...	128 to 113 in 4 mins.	51 to 39 in 4 mins.	36 to 29 in 15 mins.	23 to 17	...	241 to 228	

The "Shearing Back" of the Curves.

It will have been noticed that, owing to the extensions being plotted to an unusually large scale, the curves occupy an inconvenient amount of space. This may be avoided by adopting a geometric artifice, suggested by Professor Ewing, of "shearing back" the curves; that is, retaining the same scale of measurement, an amount is deducted from each extension proportional to the load producing it. For example, if extensions of 120, 240, 360 were produced by loads of 4, 8, and 12 tons per square inch respectively, the extensions might be diminished by, say, 100 units per 4 tons of load, and plotted as 20, 40, and 60. Another way of expressing this is to say that in the diagram the axis from which extensions are measured may simply be considered as tilted back in the manner shown below by fig. 2. Fig. 1 shows a curve drawn with ordinary rectangular axes, and fig. 2 the same curve "sheared back."

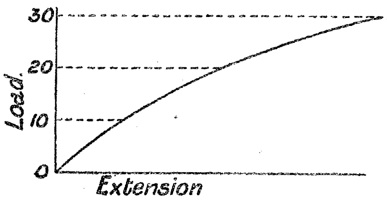


Fig. 1.

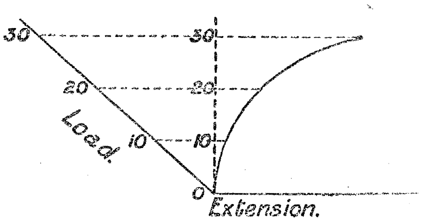


Fig. 2

Referring again to Diagram No. IIIA., the process of "shearing back" is performed graphically on the single Curve No. 6 by adopting an oblique base line, OY′, the distance from which of each point of the curve is measured horizontally and re-plotted from the vertical base line, Curve No. 6′ being obtained. Besides the convenience of space gained, this foreshortening of the curves by diminishing the rates of extension renders a more obvious comparison of similar, but slightly different, curves, and emphasises any irregularities there may be in the extensometer readings. It should thus be remembered that in a stress-strain curve which has been "sheared back" inequalities in extension are exaggerated, since the relations they bear to the total extensions are not shown.

In Diagram No. IIIB. all the curves of Diagram No. IIIA.* are shown sheared back by the method just explained, and in all similar diagrams to be shown in this paper the curves will be subjected to the same treatment. In all cases the amount of shearing of the curves is the same—the extensions are always diminished by $\frac{100}{50000}$ths of an inch for every 4 tons of stress. The scale for the measurement of extensions

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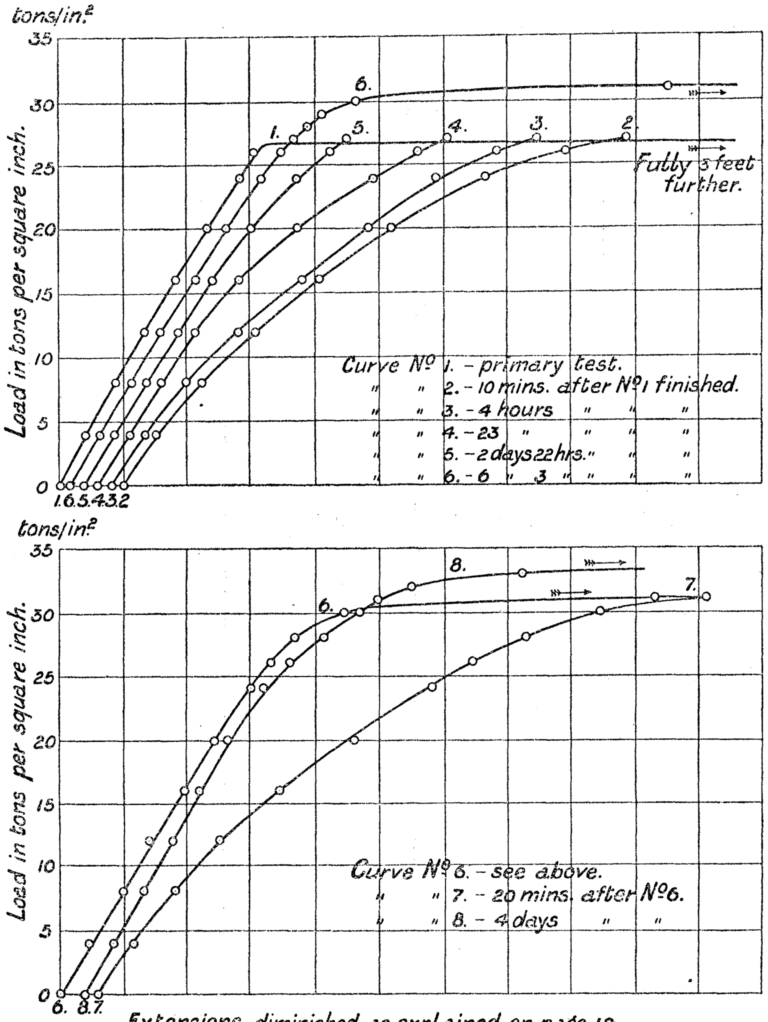
[*] Since Diagram IIIA. has been reproduced one-half full size, while Diagram IIIB. (and all other diagrams in the paper) have been reproduced to a two-thirds scale, the two Diagrams IIIA. and IIIB. are not absolutely comparable. Besides this difference due to reproduction, the scale for the measurement of load in IIIB. was originally only half that in IIIA.

will be kept the same for all analogous diagrams, but that for the measurement of load will be varied, in order to get the different diagrams suitably spaced.

Diagram No. IIIB.—(Slow recovery with time.)



Extensions-diminished as explained on page 12.

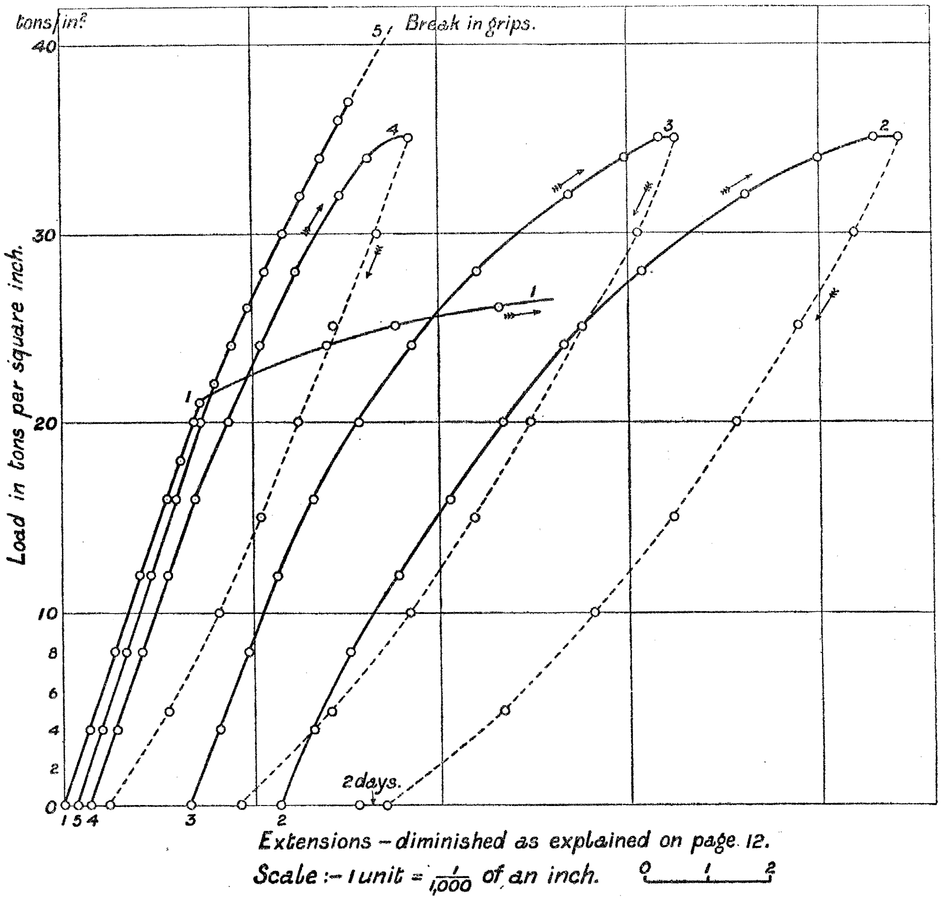
Scale:– 1 unit = $\frac{1}{1,000}$ of an inch. 0 1 2

Second Example of Slow Recovery with Lapse of Time.

In Diagram No. IV. there is shown the slow recovery from overstrain of a slightly different quality of steel under somewhat different conditions from those of the last example, while in the accompanying table the figures are given from which the various curves of that diagram have been plotted. The material is that described second on p. 4.

An examination of the differences given in the first column of the table on p. 15 will show that, although during the primary loading of the specimen, considerable yielding set in at a stress of 22 tons per square inch, it was not until almost 29 tons that such yielding as is usually associated with a yield-point occurred. Before this stress of 29 tons per square inch was reached, the stretching was not sufficient to cause the skin of oxide to spring off in the manner characteristic of a yield-point, and the removal and gradual re-application of a load of 28 tons was found to show comparatively little semi-plasticity of the material. Thus the specimen cannot be said to

Diagram No. IV.—(Slow recovery with time).



Curve No. 1—primary test of specimen.
" " 2—after load removed from No. 1.
" " 3—about 2 days after No. 1.
" " 4— " 7 " 1.
" " 5— " 17 " 1.

have been thoroughly overstrained until after 29 tons per square inch had been applied.* The primary loading was continued beyond this amount until a stress of 35 tons per square inch of original area was attained.

Curve No. 1, Diagram IV, illustrates, so far, the primary loading of this specimen.

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* A cause for the gradual manner in which this specimen yielded was perhaps, directly or indirectly, indicated by a small flaw which ran up the centre of the bar (see p. 4).

Curve No. 2 illustrates the semi-plastic condition of the material immediately after the removal of the overstraining load; while Curves Nos. 3, 4, and 5 show the progress made towards recovery, 2, 7, and 17 days respectively, after the material had been overstrained. The manner in which contraction takes place during the removal of the load is shown by dotted lines in Curves Nos. 2, 3, and 4, and it will be noticed that comparatively great retraction takes places as the lowest loads are removed. The test illustrated by Curve No. 5 shows that after 17 days' rest recovery was practically

TABLE of Readings for Diagram IV. (Slow recovery with time.)

Load in tons/in^2.	Extensometer readings for the various curves.					
	No. 1. Zero time.	Differences.	No. 2. 15 mins.	No. 3. 2 days.	No. 4. 7 days.	No. 5. 17 days.
0	0		0	0	0	0
2	60		62	61	61	60
4	121	61	129	122	122	120
6	181	60	190	188	182	179
8	241	60	259	247	242	239
10	301	60	329	309	302	299
12	361	60	398	370	363	359
14	422	61	468	433	424	419
16	482	60	539	499	485	479
18	543	61	609	568	545	539
20	604	61	681	637	612	600
21	634	30	719	669	643	630
22	684	50	755	706	674	661
23	750	66	791	740	705	692
24	810	60	830	779	737	725
25	889	79	869	815	770	757
26	998	109	909	851	802	788
27	1140	142	949	890	834	819
28	1440	300	991	930	865	850
29	—— Out of		1035	970	903	882
30	range, say 6500		1079	1011	934	915
31	...		1124	1058	967	948
32	...		1171	1104	1002	979
33	Load slowly increased to 35 tons, and then removed.		1225	1149	1036	1012
34			1279	1199	1074	1048
35			1349	1250	1133	1079
1 minute			1368	1262		
						tons/in^2.
						35½ 1096
30	...		1207	1108	980	36 1111
25	...		1037	940	820	36½ 1129
20	...		863	772	666	37 1146
15	...		689	601	510	Extensometer removed.
10	...		501	425	352	
5	...		305	237	187	Break in grips at 41 tons/in^2.
0	0·31 of an inch.		85	39	15	
Time	...		6 mins. 78	6 mins. 30		
			2 days 63			

perfect, the material approximately obeying HOOKE's law up to the maximum stress of 35 tons per square inch. The load in this test was therefore increased beyond its previous maximum amount. The extensometer, however, was shortly removed for fear of a sudden break, so that the top part of Curve No. 5 (shown dotted in the diagram) was not obtained from extensometer readings. At a stress of about 41 tons per square inch the specimen suddenly broke, unfortunately in the machine grips; before this stress was reached no yield-point had been passed, or it would have been detected by a rapid falling of the horizontal beam of the testing machine.

Recovery under Stress, and Effects of Hysteresis.

Experiments were carried out to test the effect of keeping an overstrained specimen loaded, instead of allowing it to rest in an unstressed condition, and it was found that the material, whether kept stressed or unstressed, recovered at practically the same rate.

In the following table extensometer readings are given, which show the gradual recovery from overstrain of two specimens, A and B. A was kept loaded to the maximum stress employed to produce overstrain, while B was allowed to rest free from load.

TABLE comparing Recovery under Stress and Recovery when no Load was Acting.

Load in lbs./in².	Extensometer readings.					
	Immediately after over-strain.		10 days after.		40 days after.	
	A.	B.	A.	B.	A.	B.
500	0	0	0	0	0	0
10,000	137	135	121	129	129	129
20,000	286	287	269	270	265	269
30,000	446	448	426	427	410	410
40,000	610	612	587	589	560	561
50,000	795	805	759	760	717	719
55,000	901	925	842	859	795	799

This experiment was carried out during a vacation in the Engineering Laboratory of Glasgow University (Professor BARR having kindly granted the use of apparatus and laboratory), so that the conditions of experiment are somewhat different from those explained at the beginning of this paper. A 10-ton single-lever testing machine was employed, and the load applied by thousands of pounds, instead of by tons. Professor EWING's extensometer was still used. The material tested was a half-inch rod of fairly mild steel. It gave an ultimate strength of about 32 tons per square inch, with an elongation of 26½ per cent. on an 8-inch length. The primary yield-point occurred at a stress of 48,000 lbs., or about 21½ tons per square inch. It will be noticed from the table of figures that this material recovered very slowly, for even after forty days' rest recovery was by no means complete. Specimen A of the above table was further tested after about three and a-half months, and considerable imperfection of elasticity still found.

Although the table given above shows close agreement between the elastic states of the material in the two cases, there was really an interesting difference between them. This is clearly shown by Diagram No. V. Curves A and B in that diagram illustrate

Diagram No. V.—(Recovery under stress.)

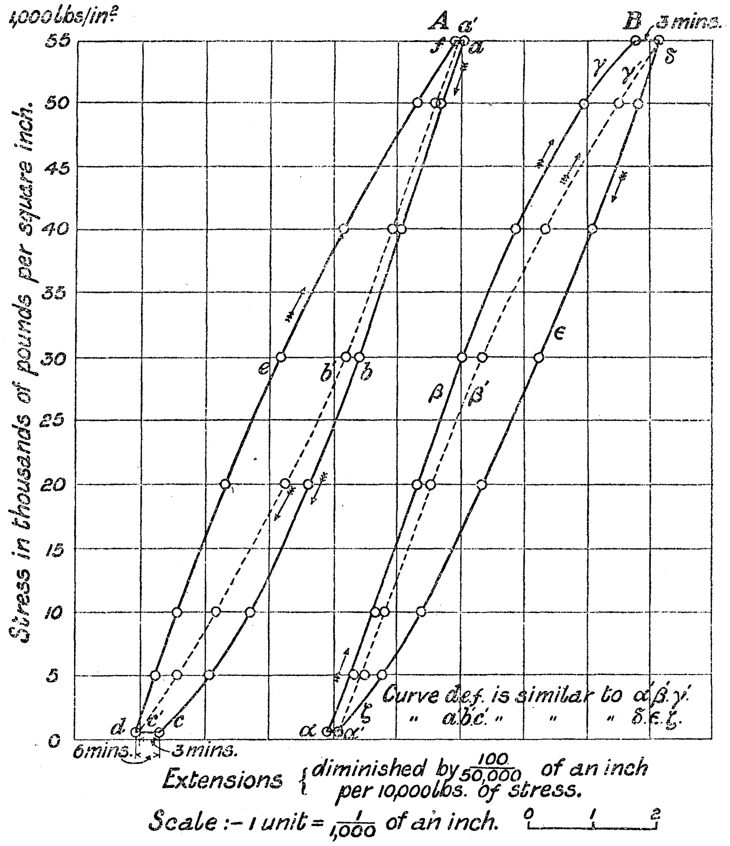


Fig. A represents the elastic condition of an overstrained specimen, which had been resting for forty days at a stress of 55,000 lbs./in^2.

Fig. B represents the elastic condition of an overstrained specimen, which had been resting for forty days free from stress.

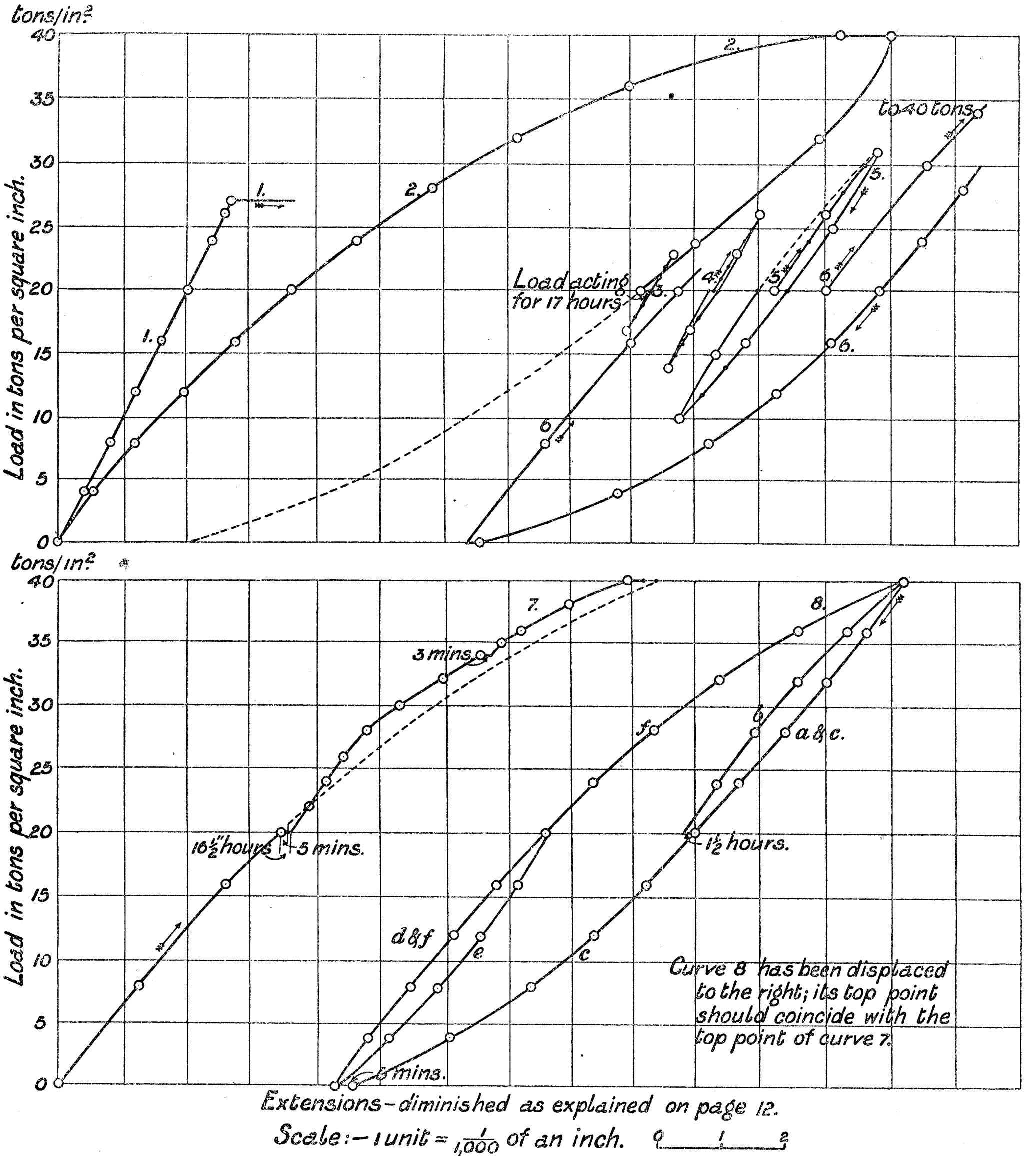
the testing of the two specimens after the forty days' rest referred to above; the last thirty of these days were uninterrupted by any intermediate testing, and during that time A was in the testing machine, with a load of 55,000 lbs. per square inch applied to it. The extensometer having been applied to A, the load was gradually removed, and readings taken, from which the part of Curve A lettered *abc* was plotted. This shows that, while a considerable proportion of the load was being removed, contraction occurred quite elastically, a straight line being first obtained at an inclination giving the Young's modulus for the material. Latterly, however, as more load was withdrawn, the retraction became more rapid, and after all load had been removed, creeping was observed to continue in a marked fashion for a few minutes, as is shown by the horizontal line *cd*. The load was now increased, and the curve *def* obtained. At the maximum stress slight creeping occurred, and then the load was once more removed, and curve $a'b'c'$ was plotted from the extensometer readings taken. This curve differs distinctly from *abc* (obtained on first removing the load), the material behaving less elastically during the early part of this second removal of the load. The curve $defa'b'c'$, however, represents an approximately cyclic state, which illustrates the imperfectly elastic condition of the material of specimen A at the time in question. When such a cycle—due to hysteresis in the relation of extension to load—is performed, work is done on the specimen, and the energy so spent is no doubt dissipated as heat.

Specimen B, which had been resting for forty days free from load, was next put into the testing machine, and the load was gradually applied. The result of the first loading is shown by the part of fig. B, Diagram V., lettered $\alpha\beta\gamma$. This curve is straight for a considerable portion ($\alpha\beta$)—the material at first approximately obeying the elastic law—but latterly greater extension occurred, and at the highest load creeping continued very obviously for a short time. The load was then removed, and curve $\delta\epsilon\zeta$ obtained. This curve resembles closely the part $a'b'c'$ of Curve A, not the part *abc*. Specimen B was next reloaded, and curve $\alpha'\beta'\gamma'$ illustrates the manner of yielding. This curve differs from $\alpha\beta\gamma$ just as $a'b'c'$ differed from *abc*. A cyclic state has now been attained, and the cycle $\alpha'\beta'\gamma'\delta\epsilon\zeta$ closely resembles that got with specimen A, which had been allowed to rest under high stress. The readings for curves *def* and $\alpha'\beta'\gamma'$ of these cycles were compared in the table given above.

It will have been noticed that it is not only the cycles ultimately obtained which are analogous in the two figures of Diagram V., but that, if one of the figures, say A, be turned upside down, then the three curves of that figure closely resemble the three curves of the other figure, B. Considering in particular the curves first obtained in the two cases, viz., *abc* and $\alpha\beta\gamma$, it will be seen that they consist of two parts. There is first a range of almost perfect elasticity, then an elastic limit is passed, and greater extension or retraction obtained, according to the curve in question. The breaking up in the structure of the material, which occurs after this elastic limit has been passed (by a decreasing or an increasing load, as the case may be), is probably analogous to the much greater breaking up which occurs on the passage of a yield-point.

In Diagram No. VI., there is illustrated the effect, on an overstrained specimen, of keeping an intermediate load acting for some time; and it will be seen that the process of recovery tends to produce an elastic range about the position of continued stress. The experiments illustrated by this diagram were carried out in the Cambridge Engineering Laboratory, hence the loading is performed in tons (not in pounds) per square inch. The material used is very similar to that employed to obtain Diagram No. III., so that the rate of recovery from overstrain is much quicker than with the material of Diagram V.

Diagram No. VI.—(Hysteresis).



Curves 4, 5 and 6 are displaced to the right—they should be continuous with one another and with Curves 2 and 3.

Curve No. 1 of Diagram VI. shows the primary elastic properties of the steel rod considered. After this first test, the specimen was largely overstrained and allowed to recover. On testing, it was then found to give a yield-point at about 40 tons per square inch, and immediately after this yield-point had been passed, Curve No. 2 was obtained.

It will be noticed from the diagram that in Curve No. 2, the removal of the load was stopped midway and the stress of 20 tons per square inch allowed to act over a night. Had the load been entirely removed, then Curve No. 2 would have continued in some such fashion as is illustrated by the dotted line in the diagram.

The continued action of the load of 20 tons was found in the morning to have produced the slight extension shown. On then testing the specimen by first increasing and then decreasing the load for a short range on either side, Curve No. 3 was first obtained, and then Curves Nos. 4, 5 and 6.

Curve No. 3 shows a short range of nearly, though not quite, perfect elasticity. A slight discrepancy of $\frac{1}{50000}$th of an inch was obtained on each side of the starting position.

Curves Nos. 4, 5 and 6 show how elastic behaviour is departed from, and greater and greater indications of hysteresis obtained as the range of loading is increased.

Curve No. 7, drawn on the bottom half of Diagram VI., shows the effect of applying a load of 20 tons to this same specimen, and allowing it to act for over sixteen hours before proceeding with the loading. That is, the effect produced by a prolonged pause in the loading of an overstrained specimen is shown. After the pause, the curve starts off at a much steeper gradient; but shortly it falls back again to a rather less inclination than if there had been no interruption in the loading. The dotted line in Curve 7 shows the manner in which the curve would have continued had no pause occurred in the loading. This continuation was accurately known; for previously the specimen had been loaded to 40 tons, three times in succession, and the last two applications had given very accurately the same curve—no two readings differing by more than $\frac{2}{50000}$ths of an inch. At the stress of 34 tons, in Curve No. 7, the effect of a three minutes' pause is shown to be similar to, but of course much smaller than, the effect of the long pause at 20 tons. This slight effect at the higher load may, however, be explained by simply considering that if creeping be allowed to occur at any load, then a small increase of load cannot be expected to produce so great an extension as it otherwise would.

After Curve No. 7 was obtained, the complete cycle represented by Curve No. 8 was gone through. The part of this cycle lettered *a* represents the partial removal of the load from Curve No. 7. The stress was only reduced to 20 tons, and then one and a-half hours were allowed to elapse. Slight back-creeping occurred instead of forward-creeping, such as happened in Curve No. 2. The load was then increased and Curve *b* obtained. This curve arrived very accurately at the same top point as Curve *a*. The load was then entirely removed, and Curve *c*, which coincides so far with Curve *a*, was obtained. After a pause of five minutes under no load, during which back-creeping showed itself, the load was re-applied to 20 tons (Curve *d*), decreased to zero (Curve *e*), and then increased to 40 tons (Curve *f*), which completed the large cycle of loading. The hysteresis exhibited by a cycle such as this may be represented numerically by expressing the breadth of the cycle at any stress, as a fraction of the total elongation of the specimen. If this be done, the hysteresis, in the relation of strain to stress, which recently overstrained iron has just been shown to exhibit, may be compared with that observed with ordinary material by Professor EWING in experiments on very long wires.* In Professor EWING'S experiments, the wires were subjected many times to a certain range of stress, and the extension at half the range was observed both as the load was applied and as it was removed. The latter extension was found, due to hysteresis, to be greater than the former; and the difference being expressed as a fraction of the extension produced by the maximum load, values were obtained ranging from $\frac{1}{86}$ in the case of high carbon steel, to $\frac{1}{300}$ in the case of an iron wire in the hard-drawn state, or $\frac{1}{333}$ with mild steel wire annealed and then hardened by stretching. The hysteresis shown at half the range of stress in the cycle described above (Curve 8, Diagram VI.) is about $\frac{1}{12}$ of the extension produced by the maximum load of 40 tons per square inch.

In order to see how far the hysteresis in the relation of strain to stress exhibited by recently overstrained iron is statical in character, or how far it depends on the rate of loading, a cycle was performed allowing ten minutes to elapse after the addition of every 4 tons of stress. The only effect was to produce a series of little notches in the curve obtained, similar to the notch shown at the stress of 34 tons in Curve 7, Diagram VI. The area of the cycle was thus not appreciably affected. The time allowed after the addition of every 4 tons was ample so far as the amount of creeping was concerned, as is clearly shown by the creeping at the stress of 20 tons in Curve 7, Diagram VI. If a much longer time had been allowed to elapse, then recovery of elasticity would have taken place as in Curves 3 and 7, Diagram VI., and the question of the static character of the hysteresis would have become complicated.

The back creeping which occurs after the removal of the load from a specimen which has been several times overstrained (for example, the creeping shown to have occurred during 5 minutes in Curve 8, Diagram VI.) is not simply due to the immediately preceding loading, but to all previous loadings. It was often observed that if sufficient time were allowed to elapse after the removal of a load, the zero reading would become negative. That is, the bar would become shorter than it was before the loading was commenced—an effect which is no doubt to be ascribed to previous overstrains, and is analogous to phenomena which have been observed in the residual charge of dielectrics, and in the torsional strains of glass and other materials.

Before leaving this section of the paper, attention should be called to the close analogy between the hysteresis effects shown in Diagram VI., and the known characteristics of magnetic hysteresis in iron.†

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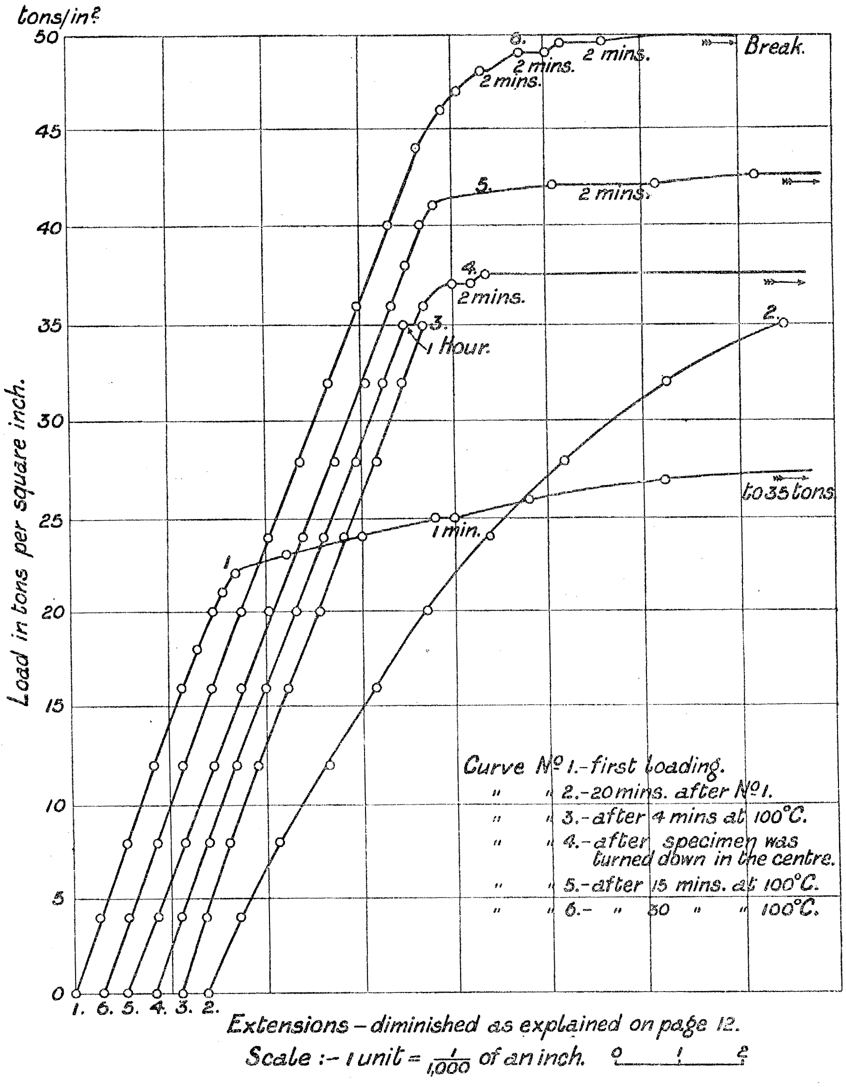
* "On Hysteresis in the Relation of Strain to Stress," 'B.A. Report,' 1889, p. 502.

† EWING. "Experimental Researches in Magnetism," 'Phil. Trans.,' 1885, or book on "Magnetic Induction in Iron and other Metals."

Effect of Moderate Temperature on Recovery from Overstrain.

The slow recovery of iron from tensile overstrain which has been illustrated by the examples already given, was found to be hastened to a remarkable extent by a slight increase of temperature. Three or 4 minutes at 100° C. were found sufficient to bring about a complete restoration of elasticity; and this, at the normal temperature, could not be effected in less than a fortnight, with the material considered.

Diagram No. VII.—(Recovery at 100° C.)



Before describing experiments which show this effect, it may be stated that experiments were made (though perhaps they might be considered unnecessary) to show that the temperatures to be dealt with could in no way alter the elastic properties of the material in its primitive condition. A virgin specimen was kept immersed in boiling water for many hours, and another was kept in a sand bath at about 250° C. for half an hour or so ; in neither case was there found, on cooling, any change in the elastic condition of the material.

In Diagram No. VII. there is shown the history of a specimen which was dipped in boiling water, whenever an overstraining load had been applied and removed. By this means recovery from overstrain instead of taking days, as in Diagram No. III., was effected in a few minutes. The material and the primary overstrain given to it are exactly as in the second example of slow recovery with lapse of time given above ; so that Curves Nos. 1 and 2 of this diagram (No. VII.) should be practically the same as Nos. 1 and 2 of Diagram IV. In order to show that the two pairs of curves are really to a close approximation identical, some of the extensometer readings taken to obtain these curves are compared in the following table. Curve No. 2 in both cases represents the first loading performed after the overstrain which is illustrated by Curve No. 1.

Load in tons/in².	Curves No. 1.		Load in tons/in².	Curves No. 2.	
	Diagram IV.	Diagram VII.		Diagram IV.	Diagram VII.
0	0	0	0	0	0
8	241	240	8	259	258
16	482	483	16	539	536
20*	604	606	24	830	829
24	810	825	32	1171	1167
28	1440	1430	35	1349	1335
29	out of range	(say 6500)			
35					

The readings for the other curves of Diagram VII. need not be tabulated.

Immediately after the readings for Curve No. 2, Diagram VII., were obtained, the specimen was taken out of the testing machine and placed in a bath of boiling water for 4 minutes. It was then removed, cooled in cold water and re-tested by gradually applying a load of 35 tons per square inch. Curve No. 3 was plotted from the observations taken. This curve is very similar to Curve No. 5 of Diagram IV., and is distinctly straighter than Curve No. 4 of that diagram ; so that 4 minutes at 100° C. has sufficed to produce more perfect recovery than would have resulted from, say, a fortnight's rest at the ordinary laboratory temperature.

After having been thus recovered and tested, the specimen was turned down in the centre to avoid breaking in the machine grips. About 4 inches at each end were left at the full diameter of 1 inch, a central portion, fully 9 inches long, being turned down to about 0·8 of an inch, and gradually tapered out at each end to the full

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* Here elastic behaviour may be said to end.

diameter. On now testing the specimen, no change was found in the behaviour of the material; Curve No. 4, which shows this test, agreeing very accurately with Curve No. 3 up to the stress of 35 tons per square inch. This maximum load of 35 tons to the square inch (now 17·50 tons total instead of 26·91 tons as before turning down) was kept on for 1 hour, with the result that slight creeping took place, as is shown in Curve No. 4. On augmenting the load a distinct yield-point was got at $37\frac{1}{2}$ tons per square inch. This second yield-point happened at a lower stress than would naturally have been expected, for in Diagram No. IV. the same material is shown to have been subjected to a stress of over 40 tons per square inch without a second yield-point having been passed. The lowness of the yield-point in the present case, was probably due to an inherent weakness in the specimen, which was shown by the small flaw which ran up the centre of the bar.* Owing to the specimen having been turned down this weakness would exert a greater influence than when the bar was of the full diameter of 1 inch. Experiments on another specimen of the same material, in fact, directly showed that after turning down a yield-point was obtained, at a stress lower than that to which the specimen had already been subjected without other than elastic yielding resulting. Such behaviour was anomalous. With other material which exhibited no flaw, turning down was found to have no effect on the position at which subsequent yielding took place.

The second yield-point shown in Diagram VII. having been passed, the material was once more in the semi-plastic state, so to effect recovery it was placed in boiling water again for 10 or 15 minutes. On cooling and re-testing, a third yield-point was obtained at $41\frac{1}{2}$ tons per square inch, as is shown by Curve No. 5, Diagram VII. The specimen was once more put in boiling water and then tested, with the result that fracture occurred at a stress of 50 tons per square inch. The break was outside the central 8-inch length, close to the tapering neck joining turned and unturned portions.

A short virgin specimen of the same rod as the above was tested (after being turned down in the centre to a diameter rather smaller than in the last case), in order to find the ordinary ultimate strength of the material. The result of this test has already been given on p. 4; local extension set in at a stress of 39 tons per square inch of original area, or about 45 tons per square inch of actual stress, and fracture was allowed to occur at that load.

The effect which a temperature of 100° C. had in hastening the recovery process was further strikingly shown by an experiment on one of the specimens employed to obtain Diagram V. This specimen had been allowed to rest for three and a-half months, and was even then found to exhibit considerable imperfection of elasticity. By heating to 100° C. for a few minutes, a marked improvement was made in the elastic behaviour of this specimen.

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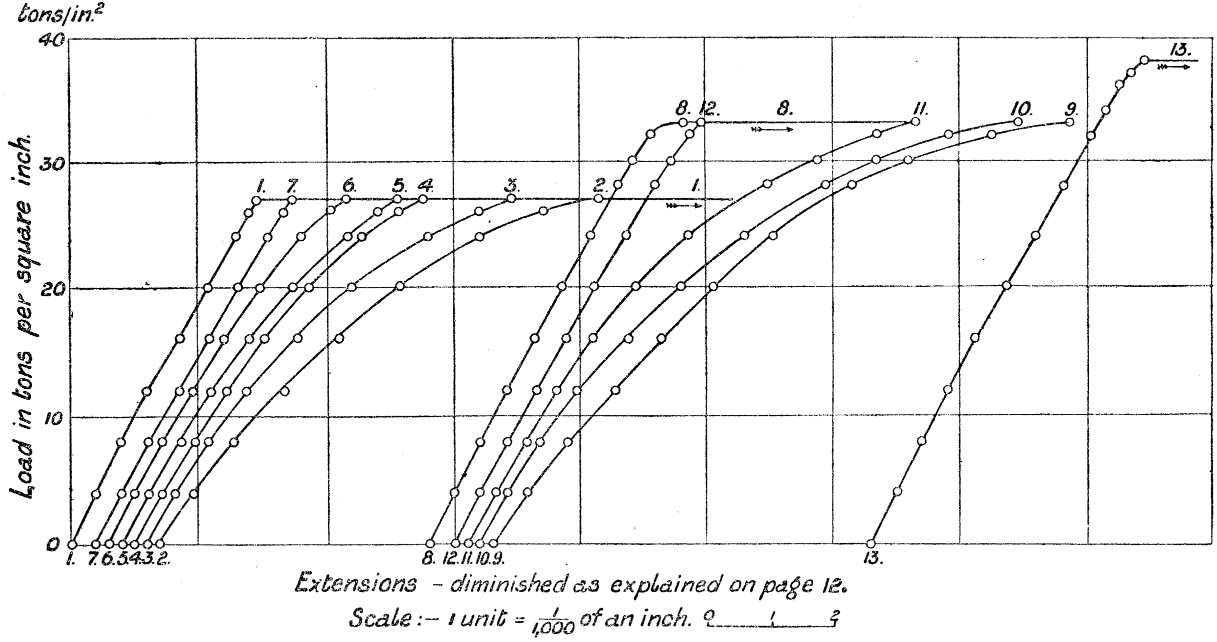
* See pp. 4 and 14.

Effect on Recovery of Temperatures below 100° C.

Diagram No. VIII., which is in two parts, gives the complete history of a specimen which was allowed to recover its elasticity at various temperatures, after having been overstrained. It shows, among other things, the very considerable hastening produced in the process of recovery from overstrain by even such a moderate temperature as 50° C. (120° Fahr.). A lengthy description of this diagram need not be given, as the side-notes accompanying the diagram give all necessary details. The tables which follow give most of the readings from which the curves of this diagram have been plotted. The material employed differed slightly from that considered last, particularly as regards the position and character of the yield-point; it resembled more closely, perhaps, the material of Diagram III.

Diagram No. VIII. (First Part).—(Recovery at 50° C., &c.)

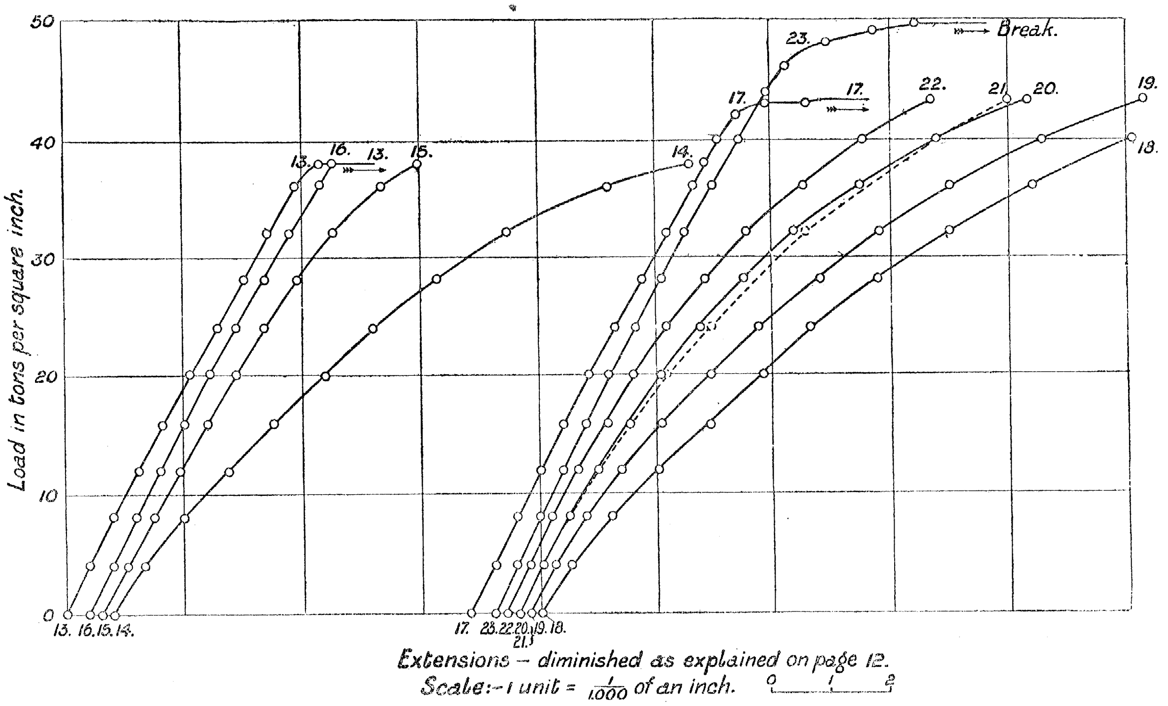


Curve No. 1.—Primary test.
,, ,, 2.—30 minutes after No. 1.
,, ,, 3.—After 5 minutes at 50° C.
,, ,, 4.— ,, 15 ,, more at 50° C.
,, ,, 5.— ,, 17 hours at normal temperature (say 13° C.).
,, ,, 6.—After 15 minutes at 50° C.

Curve No. 7.—After 5 minutes at 95° C.
,, ,, 8.—3 days after No. 7.
,, ,, 9.—20 minutes after No. 8.
,, ,, 10.—4 hours after No. 8.
,, ,, 11.—After 15 minutes at 50° C.
,, ,, 12.— ,, 15 ,, ,, 70° C.
,, ,, 13.— ,, 5 ,, ,, 95° C.

A comparison of the distances at the top between Curves 3, 4, 5, and 6, strikingly indicates the large effect of a small increase in the temperature of the restoring bath. The distance between 3 and 4 (after subtracting $\frac{1}{5}$th of a unit for change of origin), *i.e.*, $1\frac{1}{5}$ units, may be taken as a measure of the recovery due to 15 minutes at 50° C. Similarly, only $\frac{1}{5}$th of a unit (the distance between 4 and 5) measures the recovery due to 17 hours at the normal temperature (about 13° C.), while $\frac{3}{5}$ths gives the recovery due to a further 15 minutes at 50° C.

Diagram No. VIII. (Second Part).—(Recovery at 60° C., &c.)



Curve No. 13.—See first part of diagram.
,, ,, 14.—20 minutes after No. 13.
,, ,, 15.—After 15 minutes at 60° C.
,, ,, 16.— ,, 10 ,, ,, 95° C.
,, ,, 17.— ,, specimen turned down.
,, ,, 18.—20 minutes after No. 17.
,, ,, 19.—After 5 minutes at 60° C.

Curve No. 20.—After other 15 minutes at 60° C.
,, ,, 21.—After 16 hours at normal temperature.
,, ,, 22.—After 15 minutes at 60° C.
,, ,, 23.— ,, 10 ,, ,, 95° C.

Comparison of Curves Nos. 19, 20, 21, and 22, which illustrate the process of recovery after the passage of the fourth yield-point, shows a similar large difference between recovery at the ordinary temperature and that at 60° C. (140° Fahr.). In this case Curve No. 21 (obtained 16 hours after No. 20) shows that the material has yielded more, after its long rest, except for the higher loads. This apparent weakening is, of course, not due to the resting, but to the fact that the re-application of the load, necessary to obtain the readings for Curve No. 20, has had the effect of further overstraining the material to a slight extent. A curve obtained immediately after No. 20 would have fallen below that curve and also below Curve No. 21, while reaching approximately the same top point as No. 20. It may here be remarked that all the curves of this diagram have been obtained from first loadings

READINGS for Diagram No. VIII. (First Part.)

Load in tons/in².	Curve 1 (first test).	Curve 2. (30 minutes after 1.)	Curve 3. (5 minutes at 50° C.)	Curve 4. (15 minutes at 50° C.)	Curve 5. (17 hours at 13° C.)	Curve 6. (15 minutes at 50° C.)	Curve 7. (5 minutes at 95° C.)
0	0	0	0	0	0	0	0
4	119	126	122	122	122	120	120
8	239	258	248	248	247	242	241
12	360	397	377	371	370	366	366
16	486	539	517	500	499	489	489
20	608	688	660	635	633	619	611
24	729	850	820	776	777	750	734
26	789	952	910	857	849	822	798
27	820 and then off scale	1019	960 965	901 905	889 890	860	828
20	...	800	745	685	670	646	612
10	...	459	405	360	348	330	308
0	...	63	27	19	4	1	– 1

Load in tons/in².	Curve 8. (3 days after 7.)	Curve 9. (20 minutes after 8.)	Curve 10. (4 hours after 8.)	Curve 11. (15 minutes at 50° C.)	Curve 12. (15 minutes at 70° C.)	Curve 13. (5 minutes at 95° C.)
0	0	0	0	0	0	0
4	120	128	120	122	120	120
8	240	260	248	248	241	240
12	361	398	379	371	365	360
16	482	534	518	500	489	482
20	605	675	661	636	611	607
24	729	822	811	779	738	729
28	850	986	974	940	860	850
30	911	1079	1064	1029	924	910
32	978	1195	1170	1126	989	971
33	1028 and then very large yielding	1290	1250	1180	1022	1001
						tons/in².
						34 1032
20	...	869	829	761	627	36 1094
						37 1128
10	...	520	478	419	318	38 1163
0	...	99	79	35	10	and then large yielding

READINGS for Diagram No. VIII. (Second Part.)

Load in tons/in².	Curve 13. (See last table.)	Curve 14. (20 minutes after 13.)	Curve 15. (15 minutes at 60° C.)	Curve 16. (10 minutes at 95° C.)	Curve 17. (After turning down.)
0	0	0	0	0	0
4	120	128	122	120	120
8	240	260	245	240	240
12	360	398	368	360	360
16	482	536	490	480	481
20	607	678	613	602	601
24	729	820	739	726	724
28	850	973	865	849	849
32	971	1133	997	970	970
36	1094	1320	1138	1094	1092
38	1163 and large yielding	1440 } 1450 }	1219	1157	1152
					tons/in². 40 1212 42 1280 43 1330 } 1365 } 43¼ large yielding
30	...	1190	970	910	
20	...	853	662	608	
10	...	492	347	303	
0	...	78	17	−3	

Load in tons/in².	Curve 18. (20 minutes after 17.)	Curve 19. (5 minutes at 60° C.)	Curve 20. (15 minutes at 60° C.)	Curve 21. (16 hours at 15° C.)	Curve 22. (15 minutes at 60° C.)	Curve 23. (10 minutes at 95° C.)
0	0	0	0	0	0	0
4	128	122	122	122	120	120
8	261	249	247	247	240	239
12	402	380	370	371	363	360
16	547	515	498	499	488	480
20	691	657	624	629	610	600
24	832	798	759	768	741	722
28	989	950	895	907	871	845
32	1150	1100	1039	1049	1009	966
36	1320	1260	1195	1207	1158	1089
40	1508	1439	1359	1360	1308	1212
43¼	1712	1612	1518	1500	1445	...
						tons/in². 44 1336 46 1402 48 1488 49 1550 49½ 1598 and then large yielding and fracture
20	911	820	758	739	700	
0	100	38	24	19	8	

of the specimen at the various stages, so that, as explained when Diagrams Nos. V. and VI. were described, cyclic conditions of material are not represented. The difference between the behaviour of the material when a gradually increasing load was applied for a first time, and when the same load was applied for a second time, was, however, not usually so great as that shown in Curve B, Diagram V., at least with regard to the yielding at the higher loads. At early stages in recovery slightly smaller elongations were obtained on a second loading, but at intermediate stages greater extensions were obtained at the lower loads, and approximately the same extensions at the higher. The following table of extensometer readings, obtained from a specimen very similar to the last, may be taken as showing maximum differences, for the material commonly employed in these experiments, between the elongations produced at intermediate stages in recovery by a first and by a second loading. Curves Nos. 6 and 6′ of Diagram IX. also show in a striking fashion this difference in elastic condition.

Load in tons/in^2.	Extensometer readings.					
	1st application.	2nd application.	Difference.	1st application.	2nd application.	Difference.
0	0	0	0	0	0	0
4	120	120	0	120	120	0
8	246	246	0	240	246	+ 6
12	368	370	+ 2	362	370	+ 8
16	490	501	+11	488	500	+12
20	619	638	+19	618	637	+19
24	759	779	+20	760	776	+16
28	930	936	+ 6	917	925	+ 8
28½	961	959	− 2	939	945	+ 6

The effect produced by a third loading of a specimen usually differed from that produced by a second, but the difference was comparatively very slight.

To return to Diagram No. VIII., in the last test of the specimen (illustrated by Curve No. 23 in the second part of the diagram), the load was increased by a quarter of a ton to the square inch at a time; a gradual falling away from elastic behaviour was recorded, and finally local extension and fracture occurred at a stress of 49½ tons per square inch. This corresponded to about 46 tons per square inch of primitive area. The total elongation which the specimen had received was estimated to be 12 per cent. on an 8-inch length.

A fresh specimen from the same rod as the above, broken in a single test without allowing intermediate recoveries to take place,* gave an ultimate strength of rather

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* Owing to the specimen breaking in the machine grips (at a stress of 38 tons to the square inch) a partial recovery took place while the specimen was turned down in the centre. The strength given above may therefore be a little too great and the elongation a little too small.

under $40\frac{1}{2}$ tons per square inch of original area. The total elongation in this case was found to be fully 16 per cent. on the 8-inch length.

In order further to call attention to one or two features of this recovery from overstrain and the effect of temperature on it, the history of another specimen is given in Diagram No. IX.

The steel rod from which this specimen was cut differed but slightly from preceding ones. The rod was 1 inch in diameter, but the specimen was turned down, except at the ends, to a diameter of about 0·8 of an inch. The yield-point occurred at a stress of 23 tons per square inch, and was well defined like those shown in Diagrams III. and VIII., unlike those in Diagrams IV. and VII. The position of the yield-point was, however, sometimes found to vary even with specimens taken from the same rod. Thus, the specimen of the present diagram (No. IX.) gave apparently a perfectly steady extensometer reading after 22 tons per square inch had been applied—steady for, say, half a minute. The addition of the next half-ton produced rather greater elongation than was in accordance with the elastic law, but the reading was still steady. With 23 tons, however, creeping set in shortly after the extensometer reading—a rather large one—had been observed. This yielding continued, becoming greater and greater, and the skin of oxide began to spring off in the manner characteristic of the yield-point. Another specimen taken from the other end of the same bar (a 10-foot one) showed creeping and the springing off of the oxide after 22 tons of stress had been applied.

After the passage of the yield-point, illustrated in Curve No. 1, Diagram IX., the specimen was put into boiling water, and kept there for over 12 hours. This was to see if the position of the second yield-point would be affected by such prolonged treatment. As was expected, on cooling and re-testing the specimen a yield-point occurred at a load which agreed with that obtained from an adjacent specimen of the same rod, which had been immersed in boiling water for 3 minutes only. A third specimen from this same rod, after being overstrained, was put in a sand-bath, and kept at 250° C. for half-an-hour. On slowly cooling and then re-testing, the material behaved exactly as in the case of the comparison specimen, which had been restored by 3 minutes' immersion in boiling water. Had the specimen been annealed by heating to redness and slowly cooling, then, of course, the effect of overstrain would have been entirely annulled, and a yield-point obtained at a load corresponding to the stress at which the primary yield-point occurred.* It was found, however, that no effect (other than the recovery from the temporary effect of overstrain) was produced, until a fairly high temperature was attained.

To return to Diagram No. IX. After the second yield-point had been passed, the bar was re-measured and re-tested in the usual manner, Curve No. 3 being obtained. In this test the maximum load was kept on over night, and the creeping which

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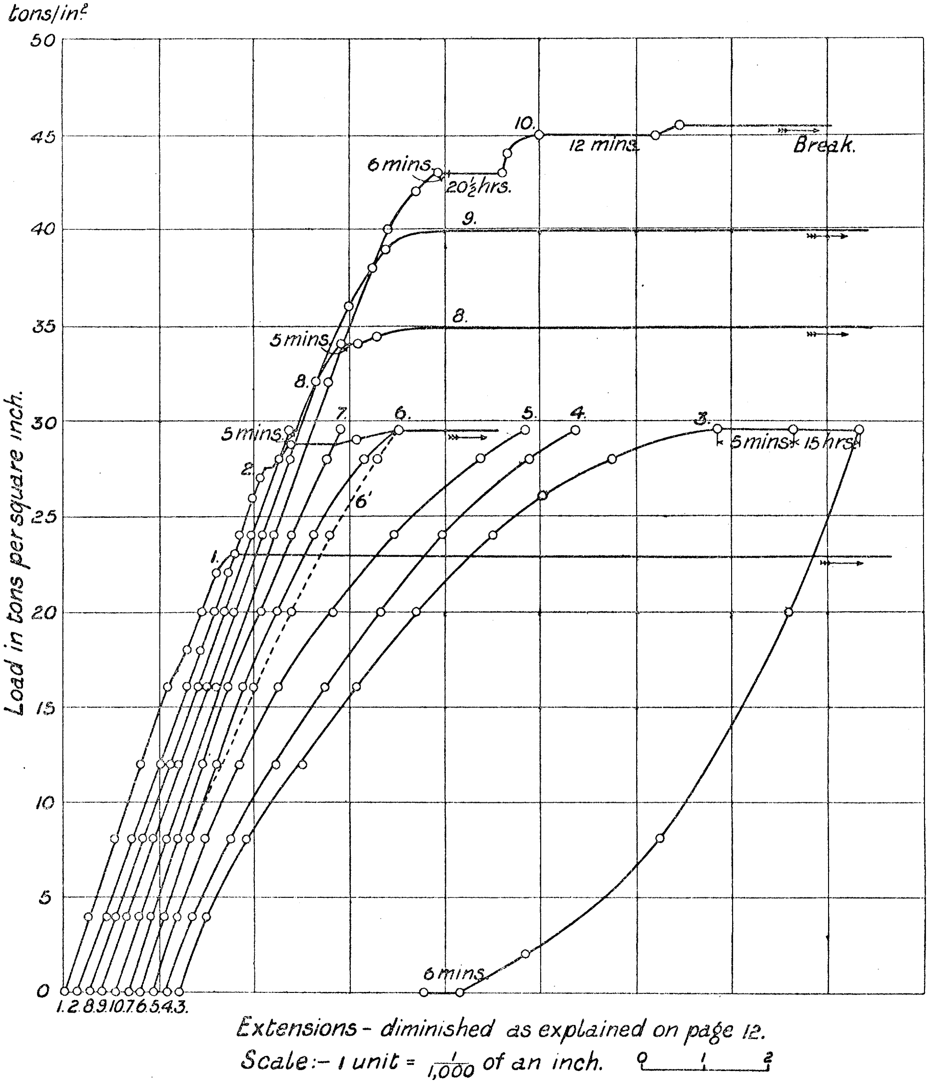
* See paper by UNWIN, "On the Yield-point of Iron and Steel, and the Effect of Repeated Straining and Annealing," 'Roy. Soc. Proc.,' vol. 57, 1895.

occurred during the first 5 minutes is shown to have been considerable; for the next 15 hours it was perhaps not so great as might have been expected. This creeping went to the production of permanent set. Curve No. 4, Diagram IX., was obtained immediately after the removal of the load from the test illustrated by Curve No. 3, and then the specimen was kept at 45° C. for 5 minutes, and afterwards for 15 minutes. The effects produced—which were slight—are shown in the following table. The

Diagram No. IX.—(Temperature effects.)



Curve No. 1.—Primary test.
„ „ 2.—After 12 hours at 100° C.
„ „ 3.—15 minutes after No. 2.
„ „ 4.—After load removed from 3.
Specimen now at 45° C., see table, p. 32.
Curve No. 5.—After 3 minutes at 60° C.

Curve No. 6.—After 4 minutes at 70° C.
„ „ 6′.—Immediately after No. 6.
„ „ 7.—After 4 minutes at 70° C.
„ „ 8.— „ 3 „ „ 100° C.
„ „ 9.— „ process A, diagram X.
„ „ 10.— „ „ B, „ X.

second column under each heading in that table gives the extensometer readings for a test performed immediately after that given in the preceding column.

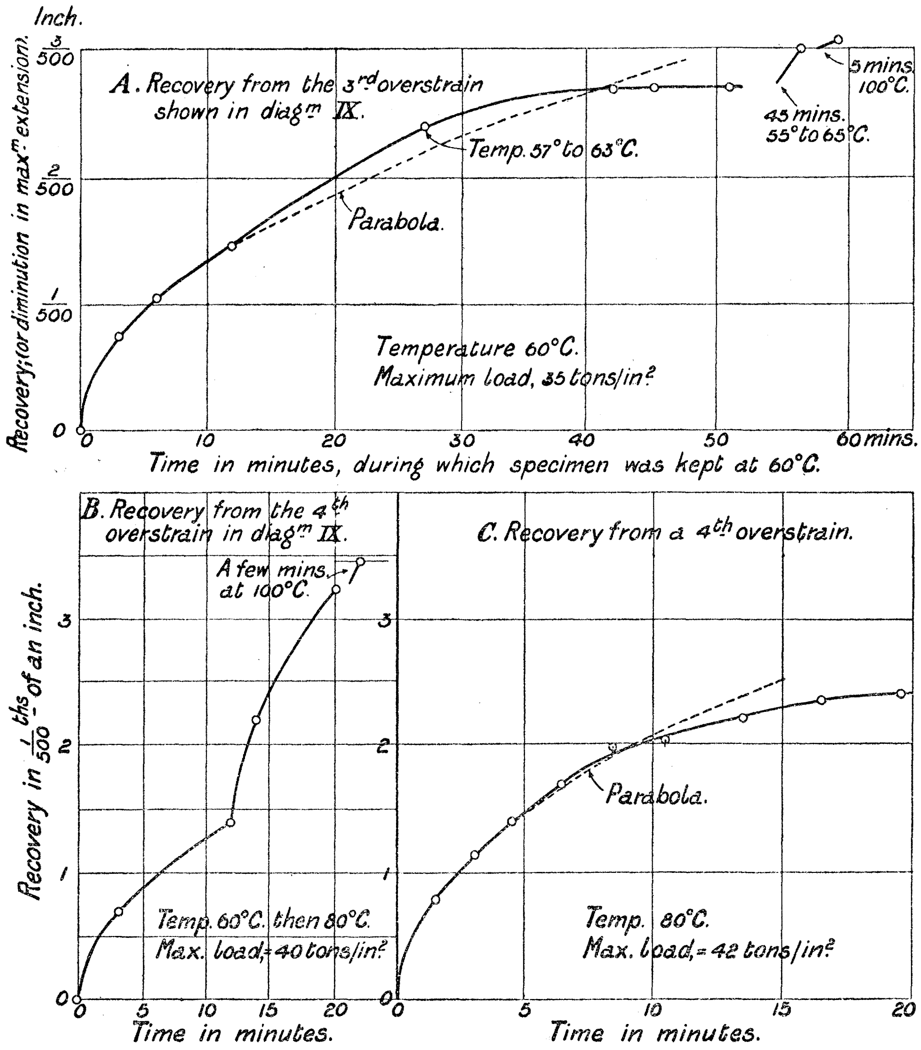
Load in tons/in².	Extensometer readings.					
	Curve No. 4, Diagram VIII.		After 5 minutes at 45° C.		After 15 minutes at 45° C.	
	1st.	2nd.	1st.	2nd.	1st.	2nd.
0	0	0	0	0	0	0
4	121	122	120	121	120	120
8	251	251	242	249	241	245
12	387	388	378	384	369	378
16	525	521	519	520	503	511
20	670	662	661	661	650	658
24	818	812	818	814	802	808
28	987	979	988	981	978	969
29½	1059	1048	1061	1049	1050	1039
¼ minute (say)	1061	1050	1070			
20	762	749	766	751	755	739
8	348	336	350	339	339	324
0	22	14	25	20	29	14
½ minute (say)	19	10	...	...	20	

These figures show that an immediate re-application of the load has produced in each of the three cases less total elongation than the first application, in consequence of the bar's gradual settlement into a cyclic state through successive loadings. After the recovery has become fairly perfect, this considerable diminution in the total elongation obtained by a second application of the testing load does not occur. This was clearly shown in the table on p. 29. In the present diagram Curve No. 6′ (shown dotted) was obtained immediately after No. 6, and it shows almost no change in the total elongation produced.

The tests made immediately after treatment at 45° C. are shown by columns 2 and 3 of the table above to have given slightly greater total elongations than those obtained from the loadings performed immediately before warming. This is perhaps contrary to what might have been expected, since increase of temperature has been shown to hasten recovery. But although the total elongations are greater, the process of recovery really has been aided. This is shown by the fact that distinctly smaller yieldings are obtained with low loads after the bar has been heated to 45° C. But though the bar is more perfectly elastic under low loads, under higher ones the yielding which occurs has not been decreased ; so that, as the specimen is subjected to a gradually increasing load, there is a transition from more elastic behaviour to less, which is suggestive of a yield-point.

To return again to Diagram No. IX., the specimen, after being warmed to 45° C. in the manner just explained, was subjected to the treatment recorded in the notes accompanying the diagram, and finally, as shown by Curve No. 8, complete restoration of elasticity was effected. The load was therefore increased until a yield-point was obtained at a stress of about 35 tons per square inch. The recovery from the overstrain produced by the passage of this third yield-point is shown by means of a curve at A in Diagram No. X. This curve was obtained by plotting amounts of recovery—

Diagram No. X.—(Time-recovery Curves.)



measured in the manner described on page 25 (that is, by the diminutions in the extensions produced by the maximum load)—against the time taken at 60° C. to produce the recoveries. This method of measurement is of course faulty, for it has been shown above that a slight recovery may have occurred, although a greater total elongation has been obtained. But if the recovery is tolerably rapid, the method may be justified, for the sake of comparing the rates of recovery at different stages of completion, by means of a time-recovery curve. At C, Diagram X., there is shown a curve of this kind obtained from a specimen whose history will not be given. This curve was more fully and carefully determined than those obtained from the specimen of Diagram No. IX.; but any of the curves in Diagram No. X. show that in the earlier stages the amount of recovery is approximately proportional to the square root of the time.

The heating of the specimens was accomplished by immersing in a hot water-bath at the required temperature for the required time, and then cooling by at once dipping in cold water. Had the specimen been allowed to cool slowly in the air, then greater recovery, due to a long and indefinite time at lower temperatures, would have been obtained.

Curve A, Diagram X., shows that at 60° C. a long time would have been required to produce perfect recovery, so the specimen (of Diagram IX.) was finally put in boiling water for 5 minutes. After cooling, a gradually increasing load was applied, and a fourth yield-point obtained at a stress of 40 tons per square inch. This test is shown by Curve No. 9, Diagram IX.

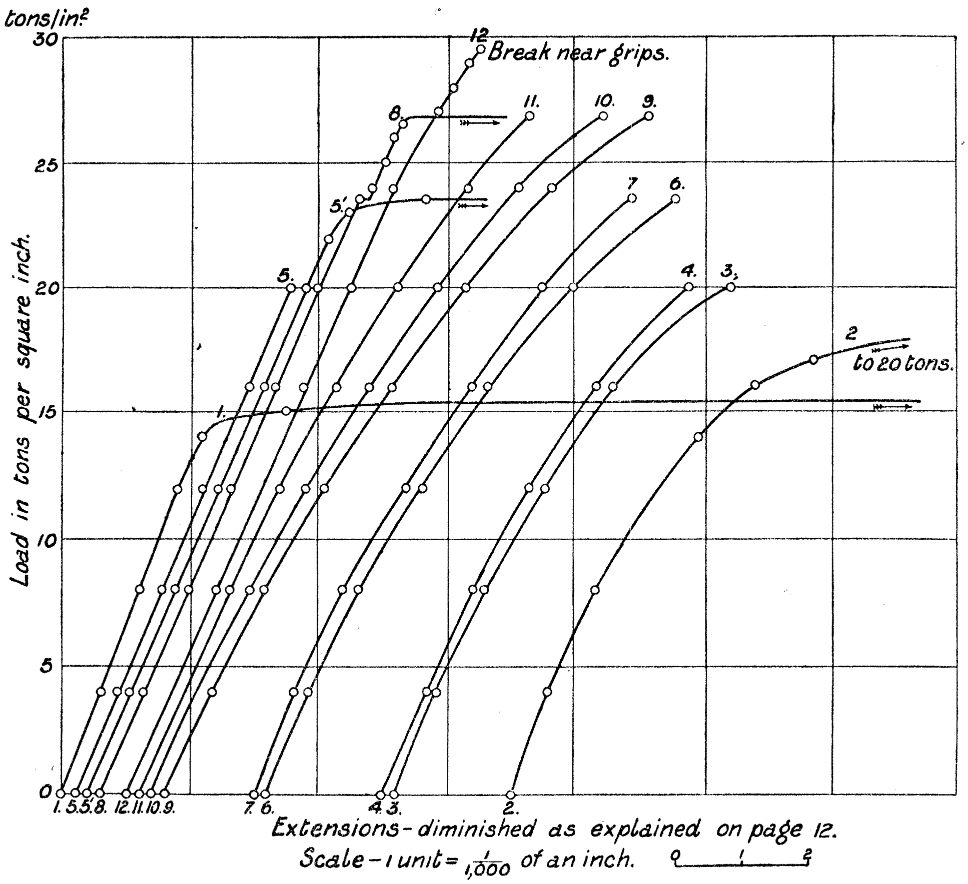
The recovery from this fourth overstrain is illustrated by Curve B, Diagram X. First 60° C. and then 80° C. were employed, perfect recovery being again obtained by bringing the piece to 100° C.

On load being once more applied to this specimen, elastic behaviour was shown up to the stress of 40 tons per square inch. At the 43rd ton, creeping was detected, and this load was allowed to remain on for 20½ hours. Considerable extension resulted, as is shown by Curve 10, Diagram IX. On now further increasing the load, the yielding was found for the subsequent three half-tons to be in close accordance with the elastic law. With the fourth half-ton (*i.e.*, at 45 tons per square inch) creeping was again detected. After 12 minutes, however, this creeping became very slow, so another half ton was applied, with the result that local extension and fracture occurred at that load of 45½ tons per square inch. This stress was equivalent to rather over 42½ tons per square inch of primitive area; the total elongation was about 0·81 of an inch, or rather over 10 per cent. on the 8-inch length.

A virgin specimen from an adjacent portion of the same bar as the above, gave, when tested at once to breaking, an ultimate strength of 36½ tons per square inch of original area; the total elongation in this case was 1·82 inches or nearly 23 per cent. on the 8-inch length.

It will be noticed that the distance in tons between the successive yield-points shown in Diagram IX. is roughly constant, and further that fracture has occurred where a yield-point (if not a fracture) would naturally have been expected. More correctly, it is the distance between a yield-point and the previous maximum load that is the same throughout. Thus a specimen from the same bar as that employed for this diagram, No. IX., was overstrained primarily by 27 tons to the square inch, and the subsequent yielding was obtained at 33 tons. That is at about 4 tons higher than the second yield-point shown in Diagram IX., when the primary loading was only carried to 23 tons per square inch. This regularity in the raising of the yield-point was also shown in Diagram No. VIII., the material being slightly different from that which has just been considered; and, further, it will be shown in Diagram No. XI., which gives the history of a specimen of unhomogeneous wrought iron. The distance between the yield-points is 3 to 3½ tons with the common wrought iron and 5 or 6 tons with the semi-mild steel usually employed in these

Diagram No. XI.—(Common iron.)



Curve No. 1. —Primary test.
,, ,, 2. —Immediately after No. 1.
,, ,, 3. — ,, ,, ,, 2.
,, ,, 4. — ,, ,, ,, 3.
,, ,, 5. —16 hours after No. 4.
,, ,, 5'.—After a few minutes at 100° C.
,, ,, 6. —Immediately after No. 5.

Curve No. 7.—Immdiately after No. 6.
,, ,, 8.—After a few minutes at 100° C.
,, ,, 9.—Immediately after No. 8.
,, ,, 10.—¼ hour after No. 9.
,, ,, 11.—After 5 minutes at 55° C.
,, ,, 12.— ,, 10 ,, ,, 100° C.

experiments. In Diagram No. VIII. yield-points were obtained at loads of about 27, 33, 38, 43¼, and 49½ tons per square inch,—fracture occurring at the last mentioned stress. With another specimen from the same steel rod the primary loading was carried to 30 tons per square inch, and after recovery of elasticity a 2nd yield-point was obtained at about 35 tons per square inch. On again restoring elasticity a 3rd yield-point was found to occur at a stress somewhat under 40 tons, and when recovery from overstrain had once more been effected fracture took place at 45 tons per square inch. It is however probable since this material is the same as in Diagram VIII., that had the primary loading in the present case been carried only to 29 tons per square inch, then a yield-point would have been obtained at a stress of 44 tons, and fracture would not have taken place until a load of over 49 tons per square inch had been applied. A yield-point obtained at a high stress is thus a crisis in the history of the specimen under test; the material is in danger of giving way, but if it does not, then, after recovery it will stand, before fracture occurs, a stress 5 or 6 tons higher than that at the critical yield-point.

It should, perhaps, be pointed out that in Diagram No. VII. no uniformity exists in the position of the yield-points. In this case the specimen cannot, perhaps, be taken as illustrating the behaviour of a certain material, for it will be remembered that a small flaw ran through the centre of the bar from which this specimen was taken, and probably this flaw had a considerable influence in determining the position of the yield-points.* Chemically this material differed only slightly from that of the other steel rods used, as is shown by the analyses given on page 4.

Before concluding this section of the paper, attention should perhaps be directly called to Diagram No. XI., which has already been incidentally referred to. It gives the history of a specimen of common wrought iron, the diameter of the specimen being 1 inch. Curve No. 1 illustrates the primary loading and shows that the yield-point has occurred at a stress of 15½ tons per square inch. After the large stretching had ceased, and the load had been removed, the 8-inch length of the specimen was found to have been stretched about 0·20 of an inch. On re-loading, the material exhibited comparatively little semi-plasticity, as is shown by Curve No. 2. The load was, therefore, increased until a stress of 20 tons per square inch was attained, the specimen being thereby stretched further by about a quarter of an inch on the 8-inch length. On re-testing, the curve obtained was still found to agree closely with Curve No. 2 up to the stress of 15 tons, but as the loading was now continued to 20 tons the semi-plasticity was more clearly shown. Curve No. 5 shows that a night's rest at the ordinary temperature has been sufficient to produce complete recovery of elasticity; so common iron recovers much more quickly than the semi-mild steel employed for the most part in the course of these experiments. It may be of interest here to recall that the half-inch specimens of comparatively mild steel, employed for Diagram No. V., recovered at a very much slower rate than the harder steel usually employed in these experiments.

After Curve No. 5 was obtained the specimen was put in boiling water for a few minutes to ensure perfect recovery. On testing, Curve No. 5 was repeated, and on increasing the load a yield-point was got at 23½ tons per square inch, as shown by Curve 5′. Curve No. 8 shows that a few minutes in boiling water has effected perfect recovery from this second overstrain. The maximum load of 23½ tons was kept on in this test for 45 hours, and only the slight creeping shown in the diagram occurred.

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* See p. 24.

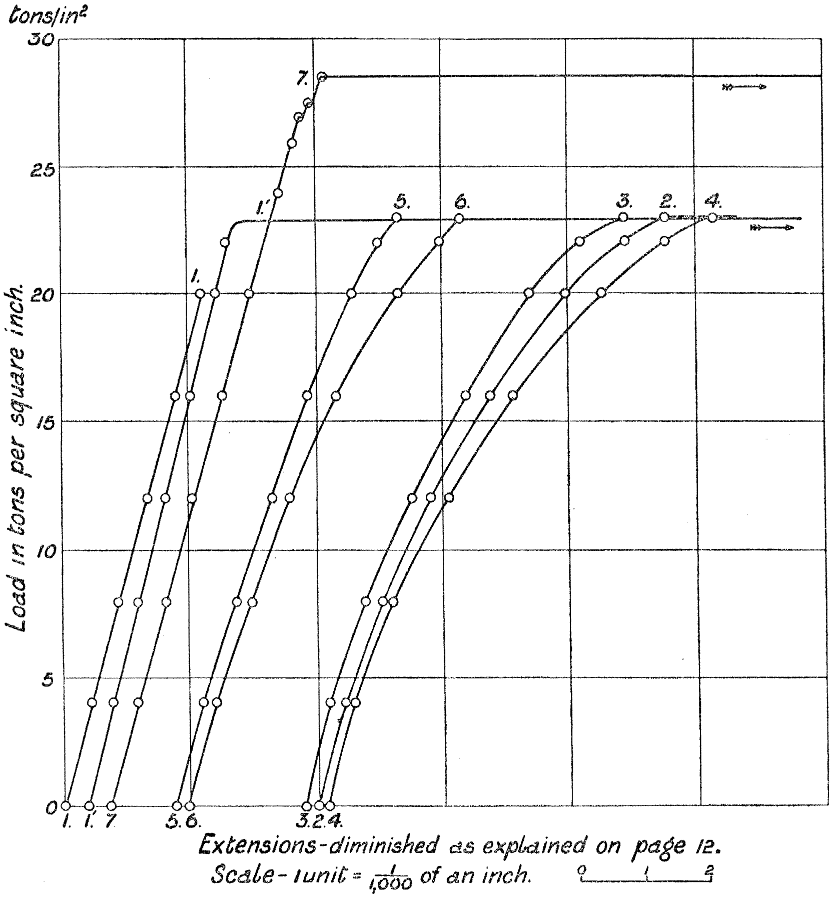
On increasing the load, a well-defined yield-point was now got at a load of 26¾ tons per square inch. Comparison of Curves Nos. 9, 10, and 11 shows the remarkable hastening in the recovery from this overstrain, produced by a temperature of 50° C.; while Curve No. 12 shows the material once more in the perfectly elastic condition. On now carefully increasing the load a fracture was obtained close to the upper machine grips, at a stress of 29½ tons per square inch. The specimen was gripped and loaded again, with the result that fracture occurred close to the lower grips at 29¾ tons per square inch. This was repeated a third and a fourth time, so that the occurrence of the fracture close to the grips was not due to a primary effect of the gripping, that is, it was not due to the gripping having prevented the material from becoming hardened by overstrain. The breaking load, of over 29½ tons per square inch, was equivalent to a stress of fully 27 tons per square inch of the original area of the specimen. Another specimen of the same rod was found to give a yield-point at 14 tons per square inch, and on steadily increasing the load fracture occurred, near the centre of the specimen, at slightly under 23 tons per square inch of original area. The elongation was about 21 per cent. on an 8-inch length. Common iron thus exhibits the same features as steel in respect of recovery from overstrain and the effect of temperature on it; but in the case of common iron recovery is comparatively rapid.

The Effect of Mechanical Vibration on Recovery from Overstrain.

Diagram No. XII. illustrates the effect of mechanical vibration on recently overstrained iron, and shows that such treatment has an opposite effect to that of increase of temperature—instead of the recovery process being hastened, the material is made distinctly less elastic. The following table gives most of the figures from which various curves of this diagram have been plotted. The material employed is the same as that used for Diagram No. IX., but the specimen in this case was not turned down, and so was of the full diameter of 1 inch throughout its length.

Load in tons/in^2.	Curve No. 1. (2nd—after vibrating.)		Curve No. 2.	Curve No. 3. (½ hour's rest.)	Curve No. 4. (After vibrating.)	Curve No. 5. (16½ hours' rest.)	Curve No. 6. (After vibrating.)
	1st	2nd				(2nd loading)	
0	0	0	0	0	0	0	0
2	60	60	61	60	59	60	60
4	122	120	122	120	119	121	120
6	182	181	188	187	185	185	185
8	245	240	251	249	251	248	249
10	307	300					
12	368	361	390	387	396	374	379
14	429	421					
16	489	482	539	529	545	502	518
18	549	544					
20	609	601	699	680	718	639	665
22		659	797	771	818	709	749
23		Off the scale	851 1008 } 10 mins.	825 840 } 3 mins.	868 919 } 10 mins.	745 750 } 3 mins.	790

Diagram No. XII.—(Mechanical vibration.)



Curve No. 1.—Illustrates primary loading.
,, ,, 1′.—Is after mechanical vibration.
,, ,, 2.—Immediately after No. 1′.
,, ,, 3.—$\frac{1}{2}$ hour after No. 2.

Curve No. 4.—After mechanical vibration.
,, ,, 5.—A 2nd loading, $16\frac{1}{2}$ hours after No. 4.
,, ,, 6.—After mechanical vibration.
,, ,, 7.— ,, 4 minutes at 100° C.

Before the experiment corresponding to Curve No. 1 was performed, a test was made to ensure that such vibration as was contemplated would have no effect on the elastic properties of the primitive material. The specimen was loaded till a stress of 20 tons per square inch was attained, and the load was then removed. The extensometer readings obtained are shown in the first column of the table given above. The specimen was then taken out of the testing machine and vigorously tapped with a hammer, so as to make it ring in various modes. It was then re-tested and the second column of readings shown above was obtained. These readings are slightly less than those obtained during the first loading, but this was to be expected on a second loading, though, perhaps, to scarcely so great an extent. Large yielding occurred during this test at 23 tons per square inch, which is the known yield-point of the material. Hence violent vibration may be said to have had no effect on the primitive material, or if it had a slight effect, it was shown in the annihilation of the causes of small departures from accurate obedience to the elastic law.

Curve No. 2 of Diagram No. XII. shows the specimen to be in the ordinary semi-plastic condition produced by overstrain, and Curve No. 3 the condition of the material after half-an-hour's rest. After this test the specimen was taken out of the testing machine and vigorously tapped with a hammer. On re-testing Curve No. 4 was obtained, which shows that not only has the effect of the half-hour's rest been annulled by the vibration, but that the material was rather more plastic than it had been immediately after overstrain. The specimen was next allowed to rest for 16½ hours and was then re-tested, Curve No. 5 showing the progress made towards recovery. The specimen was then taken out of the testing machine, and once more struck with the hammer so as to make it ring. On again testing, the elasticity was found just as before to have been made more imperfect, Curve No. 6, which illustrates this test, lying below No. 5. The specimen was then put in boiling water for a little, and Curve No. 7 shows that recovery was complete. Hammering was found to have no appreciable effect on the elastic condition of material whose elasticity had been thus restored.

In concluding this section it may be of interest to state that the effect of turning down the diameter of a recently overstrained specimen was to produce partial recovery of elasticity. This was in all probability due to the warming which accompanied the cutting action—the bar being heated by conduction, and only the surface subjected to severe mechanical vibration.

The Influence of Magnetic Agitation in Hastening or Retarding the Recovery of Elasticity.

The experiment which is now about to be described was made with the object of finding the effect on recovery, of magnetising and de-magnetising an overstrained specimen.

A coil (1¼ inch diameter × 7½ inches long) was made which gave a field strength at the centre of about 140 C.G.S. units, when a current of 10 amperes was passing. This coil was put round a specimen and supported at the 8-inch length, to which the extensometer was to be applied.

The material used was the same as that of Diagrams IV. and VII.; the specimen, however, was not in its virgin condition, it had been largely overstrained and had recovered its elasticity again, so that a yield-point was not expected till a stress of about 40 tons was attained. During the loading of the specimen, a current of 10 amperes was passed at intervals through the coil, and it was found that the extensometer could clearly detect (when the current was passed) the slight elongation due to magnetisation. This elongation occurred only at the lower loads; at the higher ones the slight contraction, which is known to occur, was quite readily observed.*

At a stress of 40¼ tons per square inch a yield-point was obtained, and while the bar was stretching rapidly at this load the current was put on and off several times, its direction being constantly reversed. The contraction, which had been noticed just before the yield-point had been reached, was still clearly shown at each "make," by

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* For the change in length caused by magnetisation, when iron is under various stresses, see papers by SHELFORD BIDWELL, 'Phil. Trans.,' A, 1888, and 'Roy. Soc. Proc.,' 1890.

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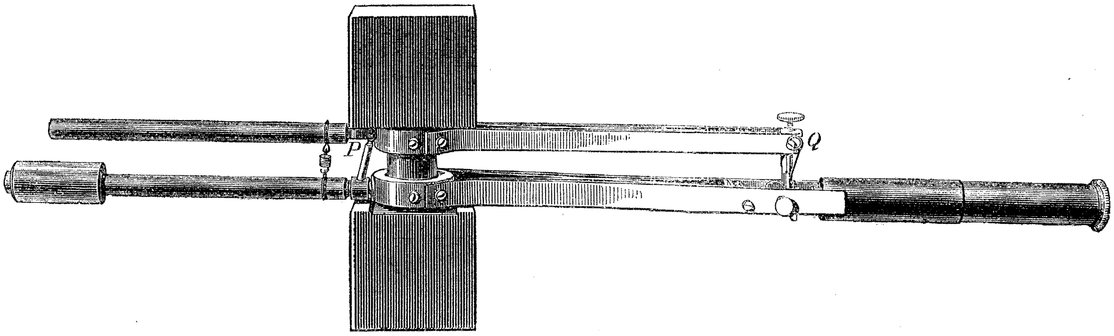
a temporary check in the rate of extension. Time readings—of a few minutes duration—taken while the bar was stretching, detected no change in the rate of extension when the current was allowed to pass for some time, and so the bar for that time kept magnetised.

When the stretching at the yield-point had practically ceased, the specimen was re-measured and the curve showing semi-plasticity obtained. The specimen was then allowed to rest for two hours, and the recovery effected was recorded by a curve. The current was next passed through the coil for periods of from 10 to 15 minutes, and was reversed all the time rapidly by hand, so that the bar was subjected to considerable magnetic agitation. Such treatment was found to have no appreciable effect on the recovery of the specimen, the curve obtained on re-testing being almost exactly the same as that obtained after the 2 hours' rest.

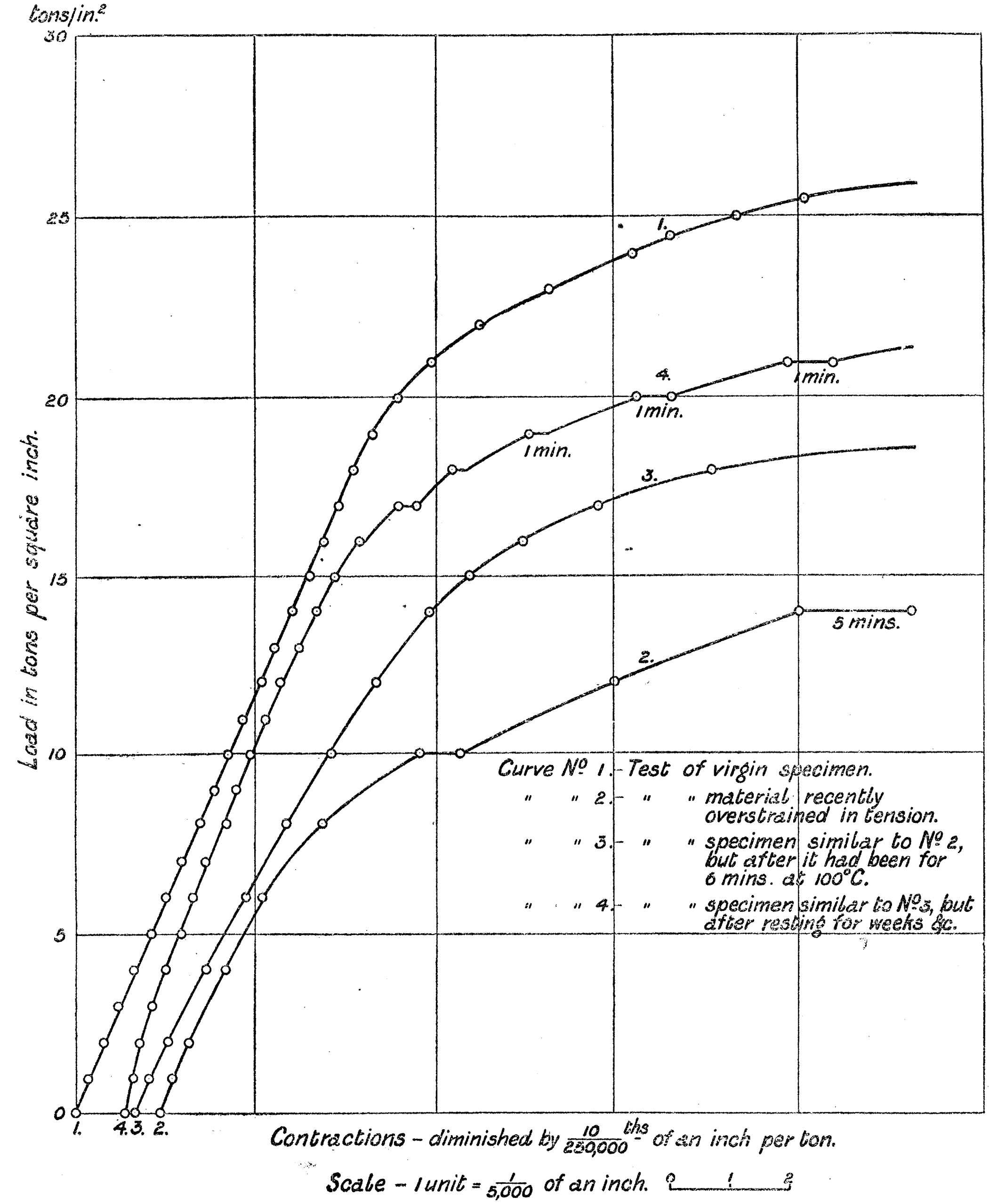
Compression Experiments.

The experiments which are now about to be described illustrate the recovery of iron from tensile overstrain by means of compression tests. These tests were carried out on small cylindric blocks, $1\frac{1}{8}$ inches diameter by $1\frac{7}{8}$ inches long, compression being applied by means of the 50-ton testing machine. The small compressional strains obtained were measured by an instrument specially designed by Professor EWING. This instrument resembles in principle Professor EWING's extensometer, especially a more recent form of that instrument, and like it is self-contained, and is entirely supported by the specimen under test. A detailed description of this instrument, which is shown attached to a compression specimen in the following illustration, need not be given here, but it may be stated that with a mechanical multiplication of 10, and further optical magnification, a contraction of $\frac{1}{250000}$th of an inch can be measured. This corresponds to a compressional strain of $\frac{1}{312500}$th, since the length of specimen actually tested is only $1\frac{1}{4}$ inches. It may be recalled that the unit of the readings of Professor EWING's extensometer represented an elongation of $\frac{1}{400000}$th, the length of specimen tested being 8 inches.



The following two series of readings, obtained with the new compression instrument, clearly show by comparison the semi-plasticity which is induced in iron by tensile overstrain. The curves which have been plotted from these readings are shown in Diagram XIII. The first series was obtained from a virgin specimen of $1\frac{1}{8}$ inch round steel rod, very similar in quality to the 1-inch rods usually employed in the tension experiments. The total length of the specimen was only $1\frac{2}{3}$ diameters, and the ends were carefully faced, so that "buckling" may be said to have been avoided, and the compressional stress applied in as uniformly distributed a manner as was practicable.

Diagram No. XIII.—(Compression experiments.)



FIRST Series of Compression Instrument Readings. (Test on a Virgin Specimen illustrated by Curve 1. Diagram XIII.)

Load in tons per square inch.	Contractions in $\frac{1}{250000}$ths of an inch.	Differences.
0	0	
1	20	20
2	43	23
4	88	45
6	134	46
8	183	49
10	227	44
12	277	50
14	321	44
16	367	46
18	410	43
20	468	58
21	505	
	time 510	
22	555	
	time 560	87
23	625	
24	705	150
25	800	
26	1000	295
	1 min. 1075	
The load was now removed and the following readings taken :—		
20	990	
15	915	75
10	815	100
5	705	110
1	622	
0	588	117

The difference column given above shows that the material has behaved elastically until a load of 20 tons per square inch was attained. Beyond that load there is shown a gradual but tolerably rapid departure from HOOKE'S law; there seems to be, however, no very definite yield-point. In a tension test of this material creeping was first noticed at 23 tons per square inch, and at $24\frac{1}{2}$ tons a very large yielding occurred. YOUNG'S modulus, as calculated from the compression readings given above, was found to agree with that obtained from tension experiments to two significant figures; in both cases the third figure was rather doubtful. Thus, the modulus as got from a first loading in tension to 20 tons per square inch, was 13,100 tons per square inch, while from a second loading to 10 tons per square inch it was found to be 13,300 tons per square inch. The modulus, as calculated from the contraction shown to have occurred in the table above, between 4 and 18 tons per square inch, is 13,600 tons per square inch.

The second series of compression instrument readings was obtained from a specimen of the same $1\frac{1}{8}$-inch rod, but after the rod had been overstrained largely in tension by a load of 33 tons per square inch. The compression specimen was cut from the overstrained bar immediately after the large stretching load was removed, care being taken to prevent warming during the cutting and mechanical manipulation necessary to the making of the small compression block. To test the effect of such mechanical treatment on the elastic condition of the material, a tension specimen was overstrained, tested, immediately turned down to a smaller diameter and tested again. As has already been recorded on page 39, considerable, though by no means perfect, recovery of elasticity was found to have been produced. Owing to the precautions taken in making the compression specimen used for this second series of readings, the mechanical manipulation may be assumed to have produced no effect on the elastic properties of the material.

SECOND Series of Compression Readings. (Material freshly overstrained, Curve 2, Diagram XIII.)

Load in tons per square inch.	Contractions in $\frac{1}{250000}$ths of an inch.	Differences.
0	0	
1	20	20
2	43	23
4	94	51
6	145	51
8	217	72
10	315	98
	time 350	
12	500	185
14	670	170
	5 mins. 768	
16	948	278
	3 mins. 1010	
18	1260	312
20	1650	
	2 mins. 1750	390
	20 mins. 1835	
Load was now removed and the following readings taken :—		
10	1615	
4	1466	
2	1415	
0	1360	

A comparison of the difference column of the present series of readings with that of the last very clearly shows the change in the elastic condition of the material, produced by tensile overstrain. There is now not even approximate conformity with HOOKE's law at the lowest loads.

The recovery of elasticity, which is brought about either by prolonged rest at normal temperatures, or by keeping the piece for a few minutes at a temperature such as 100° C., is shown in the following series of compression instrument readings. This third series of readings was obtained from a compression specimen taken from the same overstrained rod as the last; but in the present case the specimen was boiled in water for 6 minutes before being tested. This test is illustrated by Curve No. 3, Diagram XIII.

THIRD Series of Compression Readings. (Showing Recovery of Elasticity produced by 6 Minutes' Boiling.)

Load in tons per square inch.	Contractions in $\frac{1}{250000}$ths of an inch.	Differences.
0	0	
1	22	22
2	48	26
4	99	51
6	152	53
8	208	56
10	262	54
12	319	57
14	382	63
15	428 (creeping noticed)	
16	482	100
17	558	
18	660	178
19	795	
20	970	310
	1 min. 1020	
On removing the load the following readings were obtained:—		
15	912	
10	788	
4	639	
2	589	
0	540	

Comparison of the differences in this table and those in the last, or comparison of Curves 2 and 3 of Diagram XIII., clearly shows the large effect produced by the 6 minutes at 100° C. Comparison of the first and third series of readings, or of Curves 1 and 3, Diagram XIII., seems to indicate that the 6 minutes' boiling has not sufficed to produce quite perfect recovery of elasticity. In Curve No. 4 of this diagram—the readings need not be tabulated—there is shown the testing of a specimen very similar to that employed for Curve No. 3, but in this case it was certain that recovery was complete. The specimen was not taken from the same overstrained portion of a bar as that from which Curves Nos. 2 and 3 were obtained, but from another portion of the same material, which had been similarly overstrained. Before the compression specimen was cut off, the overstrained tension specimen had been allowed to rest for many weeks, and had been tested and found to be quite elastic up to a stress of 35 tons per square inch. The tension specimen was, however, boiled for some time as a further precaution, and then the compression specimen was cut from it, and the test illustrated by Curve No. 4 was performed. The modulus given by this curve agrees very well with that obtained for the virgin material from Curve No. 1. The marked discrepancy shown in this curve, No. 4, at the lowest loads may evidently be discarded; it was probably due to imperfect facing of the ends of the specimen, or some such cause.

Curve No. 4, further, very clearly shows that tensile overstrain—which raises the yield-point in tension—lowers that in compression, or, it may be more definite to say, lowers the load at which any arbitrary amount of plastic contraction occurs. This is in agreement with Professor BAUSCHINGER'S conclusion with regard to the elastic limits, viz., "that the elastic limit in tension cannot be raised without lowering the limit in compression, and *vice versâ*."* Professor BAUSCHINGER draws a further conclusion from his experiments, namely, that when the elastic limits of a material are varied by overstrain, the range of perfect elasticity seems to remain constant, so that, if the elastic limit in tension be raised, then that in compression is lowered by an equal amount. The author's experiments do not bear this out. They show that such a proposition cannot be applied to the yield-points, for the yield-point in tension of the material in the condition whose compression properties are illustrated by Curve 4, Diagram XIII., was found to occur at a stress between 12 and 13 tons per square inch, above the yield-point of the material in the primitive condition, and no matter where the yield-points in Curves 1 and 4 be supposed to exist, the lowering, which is the result of the tensile overstrain, cannot be greater than 4 or 5 tons of stress.

The characteristics of overstrained iron in respect of hysteresis and imperfect elasticity may be considered as illustrating MAXWELL'S views on the 'Constitution of Bodies,' as set forth by him in the 'Encyclopædia Britannica.'† In that article all bodies are assumed to be composed of groups of molecules oscillating about more or less stable configurations. If the oscillations are such as to cause all the groups to be continually breaking up, then we have a viscous fluid. But if "groups of greater stability are disseminated through the substance in such abundance as to build up a solid framework, the substance will be a solid, which will not be permanently deformed, except by a stress greater than a certain given stress." A solid, however, is not assumed to be entirely composed of these stable groups of molecules, or say of sensible particles, but to contain groups of less stability, and also groups which break up of themselves. When a solid has been permanently deformed or overstrained

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[*] See UNWIN'S book on 'Testing of Materials of Construction,' p. 386, or BAUSCHINGER, "Ueber die Veränderung der Elasticitätsgrenze und die Festigkeit des Eisens und Stahls," 'Mittheilungen aus dem Mech. Techn. Laboratorium in München,' 1886.

[†] Or see the 2nd volume of CLERK MAXWELL'S 'Collected Papers.'

then "some of the less stable groups have broken up and assumed new configurations, but it is quite possible that others more stable may still retain their original configurations, so that the form of the body is determined by the equilibrium between these two sets of groups; but if, on account of rise of temperature, increase of moisture, violent vibration, or any other cause, the breaking up of the less stable groups is facilitated, the more stable groups may again assert their sway, and tend to restore the body to the shape it had before its deformation."

The semi-plasticity exhibited by recently overstrained iron may thus, on the above theory, be attributed to the less stable groups, which after overstrain are in comparative abundance. And since these less stable groups will tend to break up of themselves, there will be a slow recovery through lapse of time towards elastic behaviour which is associated with the idea of stable groups.

Increase of temperature has been shown in the present paper to hasten recovery from overstrain to a remarkable extent. This, as indicated by the quotation given above, may be ascribed to a greater facility given by slight warming, to the breaking up of the less stable groups, and possibly to the re-formation of more stable groups.

Violent mechanical vibration, however, seems to break up the rather more stable groups, rendering the material more semi-plastic and hindering the recovery process.

That recovery from overstrain, or more generally, that the phenomenon of "elastic after-action," is associated with complexity in the physical structure of the material, is further borne out by the fact that a crystalline body, such as a quartz torsion fibre, exhibits little or no after-action (in the form of zero-creeping); while a complex body like glass shows such action in marked degree. An analogy to this difference in the behaviour of material, according as it is simple or complex, is found in the phenomenon of the residual charge in the Leyden jar. Condensers with pure dielectrics such as sulphur, quartz, air, exhibit little or no residual charges; while with complex substances like glass, gutta-percha, caoutchouc, the phenomenon is particularly observable.